Asymptotically safe dark matter with gauged baryon number

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Talk Outline

- Background: Renormalization group equations and asymptotic safety
  - How to find renormalization group equations
  - Evolution of couplings under the renormalization group
  - Asymptotic safety and its definition/uses
- The gauged baryon number model and its various sectors
- The UV fixed point structure
- Numerical analysis and phenomenology
- Dark matter analysis

Quantum Field Theory & Renormalization

- How are renormalization group equations (RGEs) calculated?
- Ex: Scalar ϕ^4 theory:

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \phi_0\right)^2 - \frac{1}{2} m_0 \phi_0^2 - \frac{\lambda_0}{4!}\phi_0^4 \qquad \phi_0 = \sqrt{Z}\phi_r$$

$$\mathcal{L} = \frac{1}{2} Z\left(\partial_\mu \phi_r\right)^2 - \frac{1}{2} m_0^2 Z \phi_r^2 - \frac{\lambda_0}{4!} Z^2 \phi_r^4$$

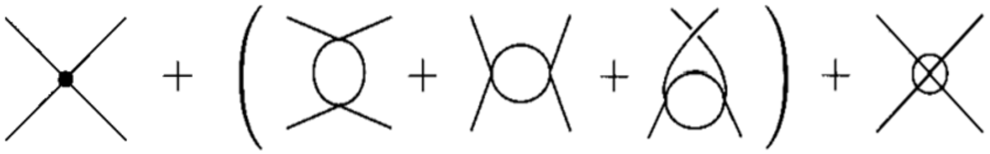
$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \phi_r\right)^2 - \frac{1}{2} m^2 \phi_r^2 - \frac{\lambda}{4!}\mu^\epsilon \phi_r^4 + \frac{1}{2}\delta_Z\left(\partial_\mu \phi_r\right)^2 - \frac{1}{2}\delta_m \phi_r^2 - \frac{\delta_\lambda}{4!}\phi_r^4$$

- μ is the renormalization scale
- $\delta_\lambda = \lambda_0 Z^2 - \lambda\mu^\epsilon$ and $\delta_m = m_0^2 Z - m^2$ and $\delta_z = Z - 1 = 0$ (at one loop)

Quantum Field Theory & Renormalization

- Consider the vertex correction:

$$-i\Gamma(p_1,p_2,p_3,p_4) =$$



- Apply dimensional regularization and $\overline{MS}$ renormalization scheme:

$$= -i\lambda + \Sigma_{b=s,t,u} \frac{\lambda^2\mu^{2\epsilon}}{2} \int \frac{d^d l}{(2\pi)^d} \int_0^1 dz \frac{1}{(l^2-\Delta_b)^2} - i\delta_\lambda \qquad \Delta_{\mathrm{b}} = m^2 - z(1-z)b^2$$

$$= -i\lambda + \Sigma_{b=s,t,u} \frac{i\lambda^2\mu^{2\epsilon}}{2} \int_0^1 dz \left[\frac{\Gamma(2-d/2)}{(4\pi)^{d/2}\Gamma(2)}\right] \left(\frac{1}{\Delta_{\mathrm{b}}}\right)^{2-d/2} - i\delta_\lambda$$

$$= -i\lambda + \frac{i\lambda^2}{16\pi^2}\mu^\epsilon \left[\frac{3}{2}\left(\frac{2}{\epsilon} - \gamma + \ln(4\pi)\right) - F(s,\mu) - F(t,\mu) - F(u,\mu)\right] - i\delta_\lambda$$

- So, $\delta_\lambda = \frac{\lambda^2}{16\pi^2}\mu^\epsilon \left[\frac{3}{2}\left(\frac{2}{\epsilon} - \gamma + \ln(4\pi)\right)\right]$

Renormalization Group Equations

$$\delta_\lambda = \frac{\lambda^2}{16\pi^2}\mu^\epsilon\left[\frac{3}{2}\left(\frac{2}{\epsilon} - \gamma + \ln(4\pi)\right)\right] = \lambda_0 - \lambda\mu^\epsilon$$

- Physical amplitudes & bare couplings are independent of choice of μ
- Apply $\mu\frac{d}{d\mu}$ to both sides to find the λ renormalization group equation at one loop:

$$\mu\frac{d\lambda}{d\mu} = \beta(\lambda) = \frac{3\lambda^2}{16\pi^2}$$

- In general, for a theory with couplings $g_1, \dots, g_k$:

$$\mu\frac{dg_i}{d\mu} = \beta(g_i)(g_1, \dots, g_k) = \sum_l \frac{1}{(4\pi)^{2l}}\beta^{(l)}(g_i) \qquad l = loop\ \#$$

Running of Couplings

- Experimentally measured values of the couplings fix the boundary conditions at an energy scale
- One of three things can happen as a coupling, g, is evolved into the UV:

1. Gaussian Fixed Point: The coupling approaches zero. Possible if $\beta(g_* = 0) = 0$
2. Interacting Fixed Point: The coupling approaches a fixed non-zero value. Possible if $\beta(g_* \neq 0) = 0$
3. Landau Pole: The coupling diverges to infinity at a finite energy scale. Field theories cannot be extrapolated past this scale

The UV Critical Surface

- UV critical surface: The set of couplings at an energy scale which run to a fixed point
  - Typically of smaller dimension than the full parameter space
- The dimension can be found by evaluating the β-functions in the vicinity of the fixed points:

$$\beta_i = M_i^j \delta_j + \mathcal{O}(\delta^2) \qquad \delta_j \equiv g_j - g_{j\star} \qquad M_i{}^j \equiv \left.\frac{\partial \beta_i}{\partial g_j}\right|_\star$$

$$M_i{}^j v_j{}^k = \vartheta_k v_i{}^k \qquad g_i(\mu) = g_{i\star} + \sum_k c_k v_i{}^k \left(\frac{\mu}{\Lambda}\right)^{\vartheta_k}$$

- UV critical surface is spanned by all v_i^k with $\theta_k < 0$ (maybe $\theta_k = 0$)

The Problem with Einstein Gravity

- It is well known that Einstein gravity as a quantum field theory is non-renormalizable

$$S_{EH} = \frac{1}{16\pi G}\int d^4x\sqrt{g}(R - 2\Lambda)$$

  - The mass dimensions of Newton's constant, G, is -2
- New couplings must be introduced to absorb divergent amplitudes at each order in perturbation theory
  - The parameter space is infinite dimensional
- The theory lacks predictive power since infinitely many measurements need to be performed to determine the values of these couplings

Asymptotic Safety

- <u>Asymptotic Safety:</u> The only points in parameter space which correspond to a physically sensible theory are those which lie on the UV critical surface
- If the UV critical surface for quantum gravity is finite dimensional, then the theory is predictive
- If this works for gravity, then couplings for other quantum fields should be asymptotically safe too
- This paradigm can reduce the dimensionality of the parameter space for a renormalizable theory

Extensions of the standard model

- Generally, extensions of the standard model contain many new underconstrained couplings. For example:
  - Kinetic mixing in $U(1)$ extensions
  - Higgs mixing angles in theories with an extended scalar sector (like two-Higgs doublet models)
  - Masses and mixing angles in theories with heavy right-handed neutrinos to utilize a see-saw type mechanism
- Asymptotic safety restricts couplings to live on the UV critical surface
  - Leads to greater predictivity and interesting phenomenology

Asymptotically Safe Gravity

- Asymptotically safe gravity is studied in a functional renormalization group (FRG) framework:

$$S_k[\phi_A] = S[\phi_A] + \Delta S_k[\phi_A] = S[\phi_A] + \int d^d q\ \phi_A R_k^{AB}(q^2)\phi_B \longrightarrow \Gamma_k[\phi_A] = \Sigma_i\ g_i(k)\mathcal{O}_i(\phi_A)$$

  - S is the "bare" action and S_k integrates out high momentum modes via the $R_k(q^2)$ functions
  - Γ_k is the effective action which depends on the momentum scale k
  - $g_i(k)$ are running couplings and $\mathcal{O}_i(\phi_A)$ are field monomials of ϕ_A and its derivatives
- For gravity, Einstein-Hilbert truncation ignores higher order terms (ex: R^2) and just considers the running of G and Λ
- The effective action evolves under the Wetterich equation: $(t = \log(k))$

$$\partial_t \Gamma_k = k\partial_k \Gamma_k = \frac{1}{2}\mathrm{STr}\left(\frac{\delta^2\Gamma_k}{\delta\phi_A\delta\phi_B} + R_k^{AB}\right)^{-1} \partial_t R_k^{BA} = \Sigma_i \beta(g_i)\mathcal{O}_i(\phi_A)$$

Gravitational Corrections

- Coupling matter to gravity alters the running of couplings for matter fields above the Planck scale. These are approximated at one loop order as:

$$\Delta\beta(g_i) = f_i g_i$$

- The form of the f_i depend on the exact matter content, gravity theory, truncation, and renormalization scheme
  - In practice, the f_i are treated as phenomenological parameters
- Gravity couples universally to matter fields:
  - f_i contributes identically to the running of all gauge couplings. True too for Yukawa & quartic couplings. So, $f_g,\ f_y,\ f_\lambda$ are input parameters
- Below the Planck scale gravity fluctuations fall off quickly. So, essentially no RGE effect for matter fields. In practice, this is modeled as:

$$\Delta\beta(g_i) = f_i\theta(\mu - M_{pl})g_i$$

Talk Outline

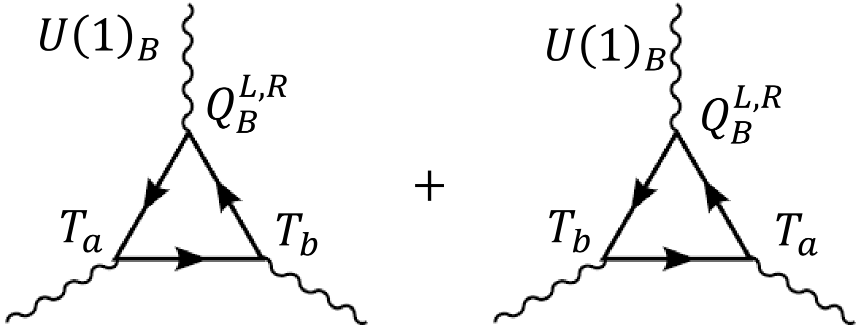
- Background: Renormalization group equations and asymptotic safety
- The gauged baryon number model and its various sectors
  - Why gauge baryon number?
  - Cancelation on anomalies
  - Particle content
  - The different sectors
- The UV fixed point structure
- Numerical analysis and phenomenology
- Dark matter analysis

Why Gauge Baryon Number?

- Baryon number is a $U(1)$ global symmetry of the standard model
- Kinetic mixing between gauged baryon number and hypercharge has interesting phenomenology for standard model particles
- Affects cosmological processes like baryogenesis and the matter anti-matter asymmetry
- Higgs mixing is possible if gauged baryon number is spontaneously broken by a complex scalar
- New fermions charged under this symmetry may provide a dark sector

Anomaly Cancellation

- This requires cancelation of anomalies by adding new fermions
  - The cancelation constrains the $U(1)_B$ charges of the new fermions
  - Assume the new fermions, $\psi_{L,R}^{\nu,e,\ell}$, are vector-like, chiral under baryon number, and charged identical to the standard model leptons
- Ex: $U(1)_B SU(2)_W^2$ anomaly $q_L \sim \left(3, 2, \frac{1}{6}, \frac{1}{3}\right),\ \psi_L^\ell \sim \left(0, 2, -\frac{1}{2}, Q_B^L\right),\ \psi_R^\ell \sim \left(0, 2, -\frac{1}{2}, Q_B^R\right)$



$$A_{ab,U(1)_B}^{L,R} = Tr[\{T_a, T_b\}]\Sigma_i Q_B^{L,R}$$

$$A_{ab,U(1)_B} = A_{ab,U(1)_B}^{L} - A_{ab,U(1)_B}^{R}$$

$$A_{ab,\,U(1)_B}^{L} = Tr[\{T_a, T_b\}]\Sigma_i Q_{B,i}^L = \delta_{ab}\left[3\left(\frac{1}{3}\right) + Q_B^L\right]$$

$$A_{ab,\,U(1)_B}^{R} = Tr[\{T_a, T_b\}]\Sigma_i Q_{B,i}^R = \delta_{ab}\left[Q_B^R\right]$$

$$A_{ab,U(1)_B} = \delta_{ab}\left[1 + Q_B^L - Q_B^R\right] = 0$$

$$\boxed{Q_B^L - Q_B^R = -1}$$

A Gauged Baryon Number Model

- 3 new generations of Dirac, vector-like fermions, ψ_i^j
- Standard model particles + right-handed neutrinos
- 1 new Dirac, vector-like fermion, χ
- 1 new vector boson, Z'
- 1 new complex scalar boson, ϕ

	$\mathrm{SU}(3)_C$	$\mathrm{SU}(2)_W$	$\mathrm{U}(1)_Y$	$\mathrm{U}(1)_B$
q_L	3	2	1/6	1/3
u_R	3	1	2/3	1/3
d_R	3	1	−1/3	1/3
ℓ_L	1	2	−1/2	0
e_R	1	1	−1	0
ν_R	1	1	0	0

	$\mathrm{SU}(3)_C$	$\mathrm{SU}(2)_W$	$\mathrm{U}(1)_Y$	$\mathrm{U}(1)_B$
ψ_L^ℓ	1	2	−1/2	= 0
ψ_R^ℓ	1	2	−1/2	= 1
ψ_L^e	1	1	−1	= 1
ψ_R^e	1	1	−1	= 0
ψ_L^ν	1	1	0	= 1
ψ_R^ν	1	1	0	= 0

$\chi_L \sim \chi_R \sim (1, 1, 0, 1/6)$

ϕ is a singlet under the standard model gauge group with baryon number +1

The Yukawa Sector

- Yukawa terms coupling the new fermions together are:

$$\mathcal{L}_y = \overline{\psi_L^\ell}\, y_1\, \psi_R^\ell\, \phi^* + \overline{\psi_L^e}\, y_2\, \psi_R^e\, \phi + \overline{\psi_L^\nu}\, y_3\, \psi_R^\nu\, \phi + \text{ H.c.}$$

- The mass scale of the new fermions is set by the ϕ vev (TeV-scale)
- Coupling heavy and light fermions together are also possible:

$$\mathcal{L}_\kappa = \overline{\psi_L^\ell}\, H\, \kappa_1 e_R + \overline{\ell_L}\, H\, \kappa_2\, \psi_R^e + \overline{\ell_L}\, \widetilde{H}\, \kappa_3 \psi_R^\nu + \text{ H.c.}$$

- This allows for decays to standard model particles
- We adopt a simplified flavor structure where y_i and κ_i are proportional to the identity matrix

The Scalar Potential

- Asymptotic safety also applies to the scalar sector:

$$V = -m_H^2 H^\dagger H + \frac{\lambda}{2}(H^\dagger H)^2 - m_\phi^2 \phi^* \phi + \frac{\lambda_\phi}{2}(\phi^*\phi)^2 + \lambda_m \phi^* \phi H^\dagger H$$

- Stability of the potential depends on the couplings:

$$V = -m_H^2 H^\dagger H - m_\phi^2 \phi^* \phi + \frac{\lambda}{2}\left[H^\dagger H + \frac{\lambda_m}{\lambda}\phi^*\phi\right]^2 + \frac{\mathfrak{s}}{2\,\lambda}\left[\phi^*\phi\right]^2 \qquad \mathfrak{s} = \lambda\lambda_\phi - \lambda_m^2$$

$$\det M^2 \equiv m_+^2 m_-^2 = v^2 v_\phi^2(\lambda\lambda_\phi - \lambda_m^2) \equiv v^2 v_\phi^2 \mathfrak{s}$$

- Following the approach of Nie & Sher (arXiv:hep-ph/9811234), we track the RGE evolution of $\mathfrak{s} > 0$ and $\lambda > 0$ to ensure vacuum stability

Kinetic Terms and Mixing

$$\mathcal{L} \supset -\frac{1}{4}B^{\mu\nu}B_{\mu\nu} - \frac{1}{4}F_Y^{\mu\nu}F_{\mu\nu}^Y + \frac{\epsilon}{2}B^{\mu\nu}F_{\mu\nu}^Y$$

- Rescale gauge fields so that gauge coupling dependence is clear:

$$\mathcal{L} = -\frac{1}{4}(G^{-2})_{AB}F_{\mu\nu}^A F^{B\,\mu\nu}$$

- Change the field bases so that the matrix G is upper triangular:

$$G = \begin{pmatrix} g_Y & \frac{\epsilon}{\sqrt{1-\epsilon^2}}g_Y \\ 0 & \frac{1}{\sqrt{1-\epsilon^2}}g_{B0} \end{pmatrix} \equiv \begin{pmatrix} g_Y & \tilde{g} \\ 0 & g_B \end{pmatrix}$$

(We will use $SU(5)$ GUT normalization of hypercharge, $g_1 = \sqrt{\frac{5}{3}}g_Y$, for the rest of the analysis)

- In this field basis, the covariant derivative takes the form:

$$D_\mu\chi = [\partial_\mu - i\,g_B B_\mu Q_B - i\,(g_Y A_\mu^Y + \tilde{g}\,B_\mu)Q_Y]\,\chi$$

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- The UV fixed point structure
  - Renormalization group equations
  - Fixed-point structure
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Renormalization Group Equations

$$\beta(c) = \sum_{\ell=1}^{\infty} \frac{1}{(4\pi)^{2\ell}} \beta^{(\ell)}(c) \qquad \hat{f} \equiv (4\pi)^2 f$$

- The Abelian gauge sector RGEs (one Loop):

$$\boxed{\beta^{(1)}(g_1) = \frac{77}{10} g_1^3 - \theta(\mu - M_{Pl}) \hat{f}_g g_1}$$

$$\beta^{(1)}(g_B) = 11 g_B^3 + \frac{77}{6} g_B \tilde{g}^2 - \frac{16}{3} g_B^2 \tilde{g} - \theta(\mu - M_{Pl}) \hat{f}_g g_B$$

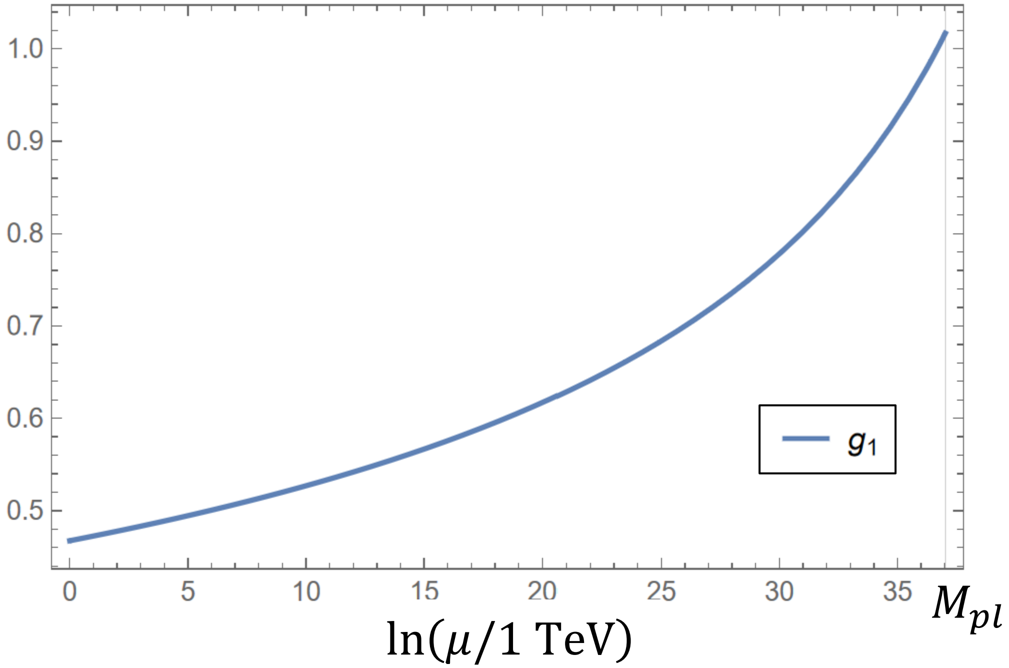
$$\beta^{(1)}(\tilde{g}) = -\frac{16}{5} g_1^2 g_B - \frac{16}{3} g_B \tilde{g}^2 + \frac{77}{5} g_1^2 \tilde{g} + 11 g_B^2 \tilde{g} + \frac{77}{6} \tilde{g}^3 - \theta(\mu - M_{Pl}) \hat{f}_g \tilde{g}$$

- Ultraviolet fixed points satisfy the conditions:

$$\beta^{(1)}(g_{1_*}) = \beta^{(1)}(g_{B_*}) = \beta^{(1)}(\tilde{g}_*) = 0$$

Running of the Hypercharge Coupling

$\mu < M_{pl}$: Continuity requires matching at Planck scale $\mu \geq M_{pl}$:



$$g_1(\mu) = \frac{g_1(\mu_0)}{\sqrt{1 - \frac{77}{10}\frac{2\,g_1^2(\mu_0)}{(4\pi)^2}\ln\left(\frac{\mu}{\mu_0}\right)}}$$

$$\beta^{(1)}(g_{1_*}) = 0 \Longrightarrow \left(\frac{77}{10}g_{1_*}^2 - \hat{f}_g\right)g_{1_*} = 0$$

$$g_{1_*} = 0 \Longrightarrow \hat{f}_g > \hat{f}_g^{crit}$$

(Gaussian Fixed Point)

$$g_{1_*} = g_1(M_{pl}) = \sqrt{\frac{10\hat{f}_g^{crit}}{77}}$$

(Interacting Fixed Point)

$$\hat{f}_g^{\mathrm{crit}} \equiv \frac{77}{10}g_1^2(M_{\mathrm{Pl}}) = \frac{g_1^2(\mu_0)}{\frac{10}{77} - \frac{2\,g_1^2(\mu_0)}{(4\pi)^2}\ln\left(\frac{M_{\mathrm{Pl}}}{\mu_0}\right)} \approx 7.9610$$

Renormalization Group Equations

$$\beta(c) = \sum_{\ell=1}^{\infty} \frac{1}{(4\pi)^{2\ell}} \beta^{(\ell)}(c)$$

$$\hat{f} \equiv (4\pi)^2 f$$

- The Abelian gauge sector RGEs (one Loop):

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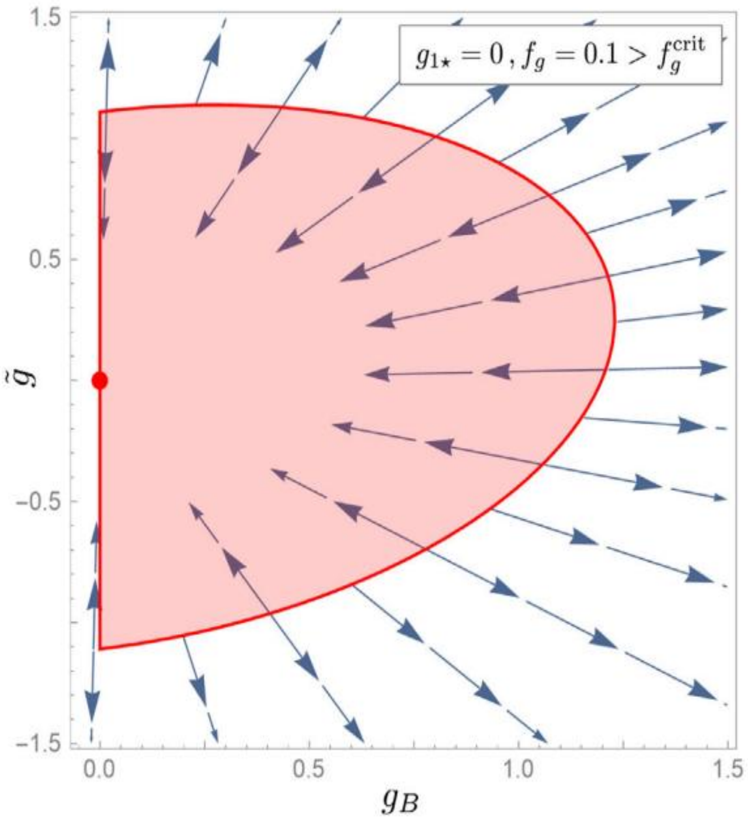
$$\boxed{\begin{aligned} \beta^{(1)}(g_B) &= 11 g_B^3 + \frac{77}{6} g_B \tilde{g}^2 - \frac{16}{3} g_B^2 \tilde{g} - \theta(\mu - M_{Pl}) \hat{f}_g g_B \\ \beta^{(1)}(\tilde{g}) &= -\frac{16}{5} g_1^2 g_B - \frac{16}{3} g_B \tilde{g}^2 + \frac{77}{5} g_1^2 \tilde{g} + 11 g_B^2 \tilde{g} + \frac{77}{6} \tilde{g}^3 - \theta(\mu - M_{Pl}) \hat{f}_g \tilde{g} \end{aligned}}$$

- Ultraviolet fixed points satisfy the conditions:

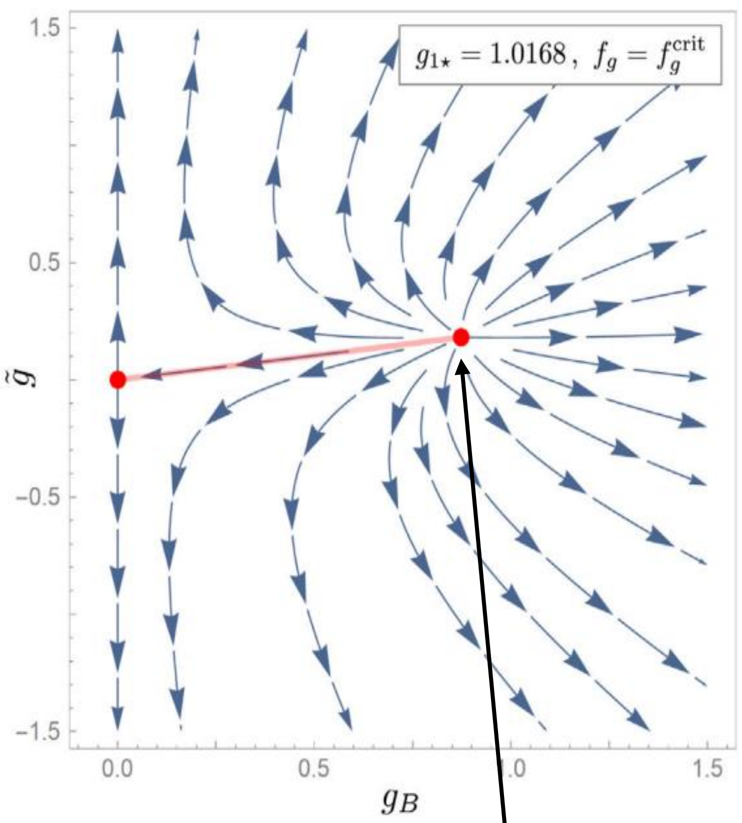
$$\beta^{(1)}(g_{1_*}) = \beta^{(1)}(g_{B_*}) = \beta^{(1)}(\tilde{g}_*) = 0$$

Ultraviolet Fixed Points

Gaussian g_1 fixed point



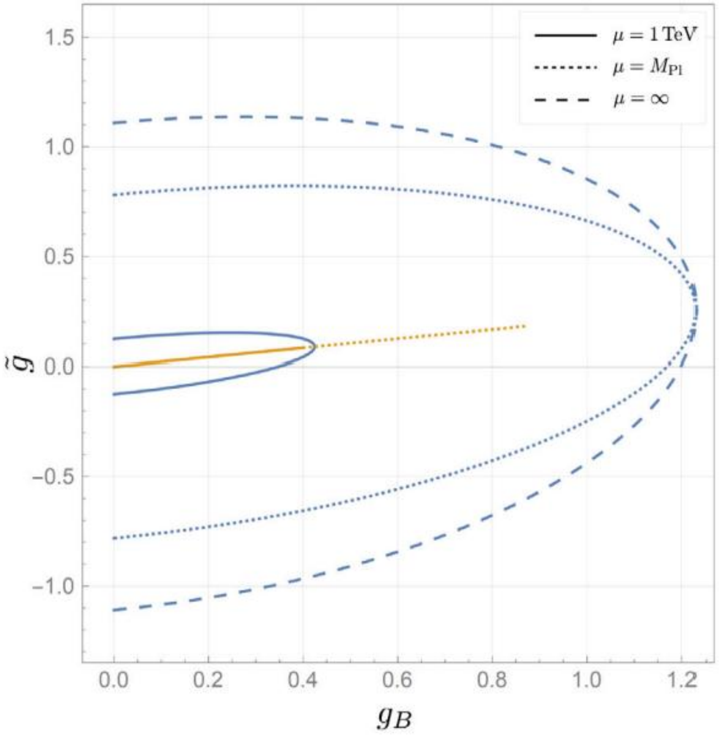
Interacting g_1 fixed point



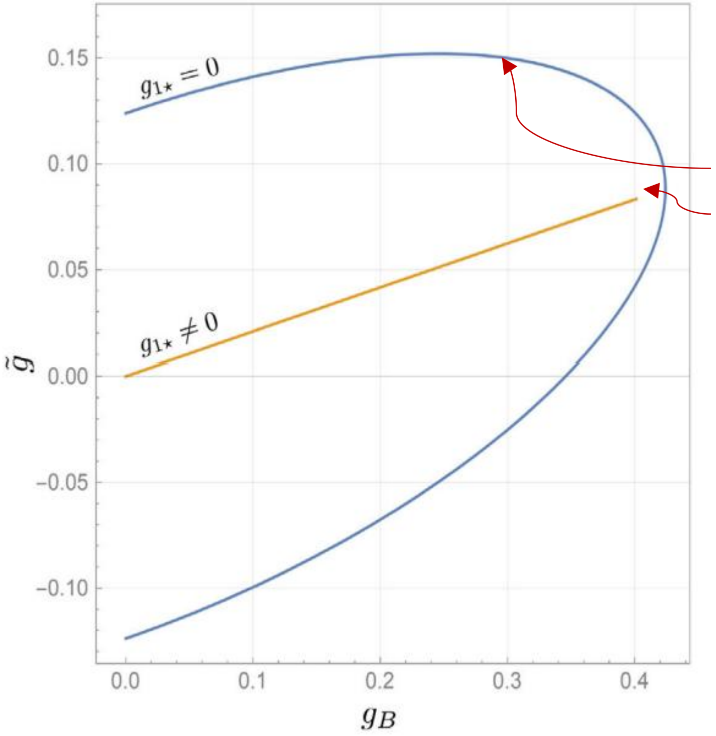
$$E(g_B, \tilde{g}, \hat{f}_g) = \frac{77}{6}\,\tilde{g}^2 + 11\,g_B^2 - \frac{16}{3} g_B\,\tilde{g} - \hat{f}_g$$

Fixed Point: $(g_B, \tilde{g}) = (0.871, 0.181)$

The UV Critical Surface



$\mu = 1$ TeV



Representative coupling values at $\mu_0 = 1$ TeV:

Case	g_B	$\tilde{g}$	m_B	f_g
$g_{1\star} = 0$:	0.3	0.14988	3 TeV	0.1 $(> f_g^{\mathrm{crit}})$
$g_{1\star} \neq 0$:	0.40128	0.08338	4.01 TeV	0.05041 $(= f_g^{\mathrm{crit}})$

Other Parameter Choices:

- $v_\phi = 10$ TeV
- $\lambda_\phi(\mu_0) = 0.2$
- $\lambda_m(\mu_0) = -0.004$
- $\kappa_i(\mu_0) = y_i(\mu_0) = 0.1$
- $f_\lambda = f_y = 0.1$

$m_h = 125$ GeV $\Longrightarrow \lambda(\mu_0) = 0.25828 \Longrightarrow m_\phi = 4.472$ TeV

- $\theta_{mix} \sim \mathcal{O}(10^{-4})$
- Current bounds are $\mathcal{O}(10^{-1})$

(J. Claude and S. Godfrey, arXiv:2104.01096)

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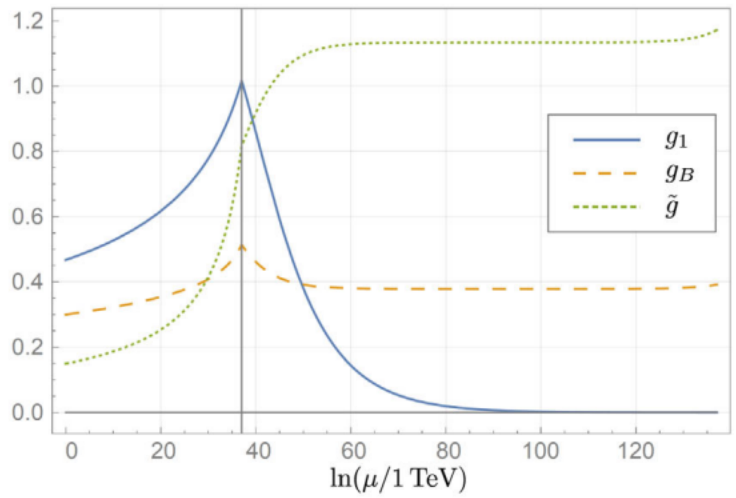
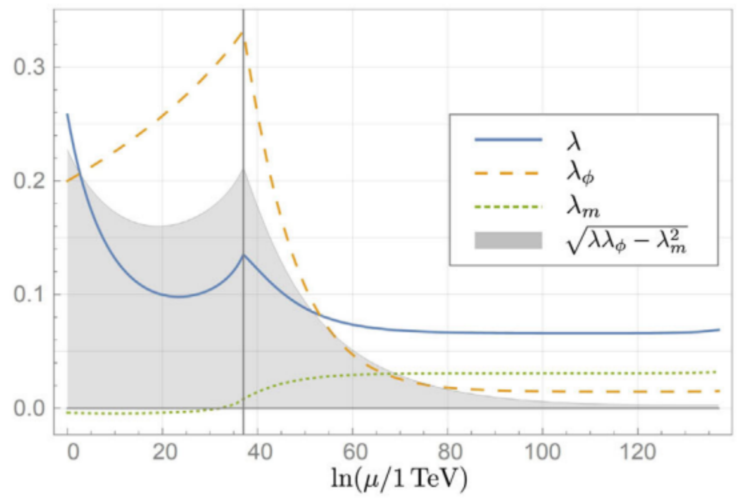
- Numerical analysis and phenomenology
  - Evolution of couplings
  - Explore the parameter space
  - Phenomenology of the baryon number gauge boson
- Dark matter analysis

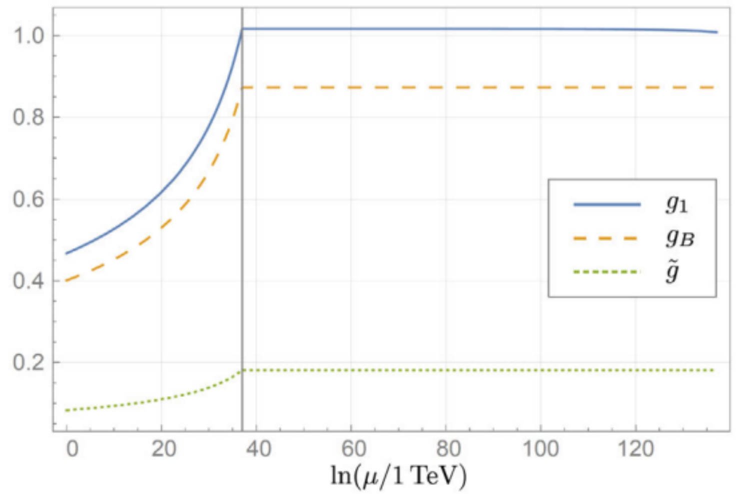
Evolution of Gauge & Scalar Couplings

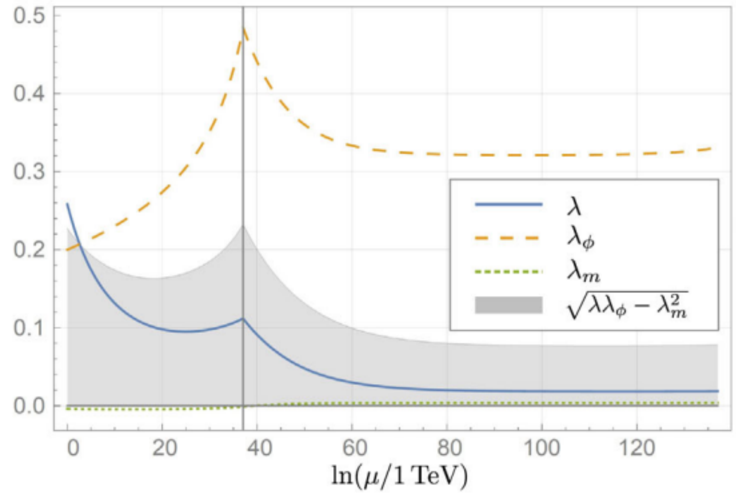




Gaussian g_1 fixed point

$$\lambda_\star = 0.065$$
$$\lambda_{\phi\star} = 0.014$$
$$\lambda_{m\star} = 0.030$$

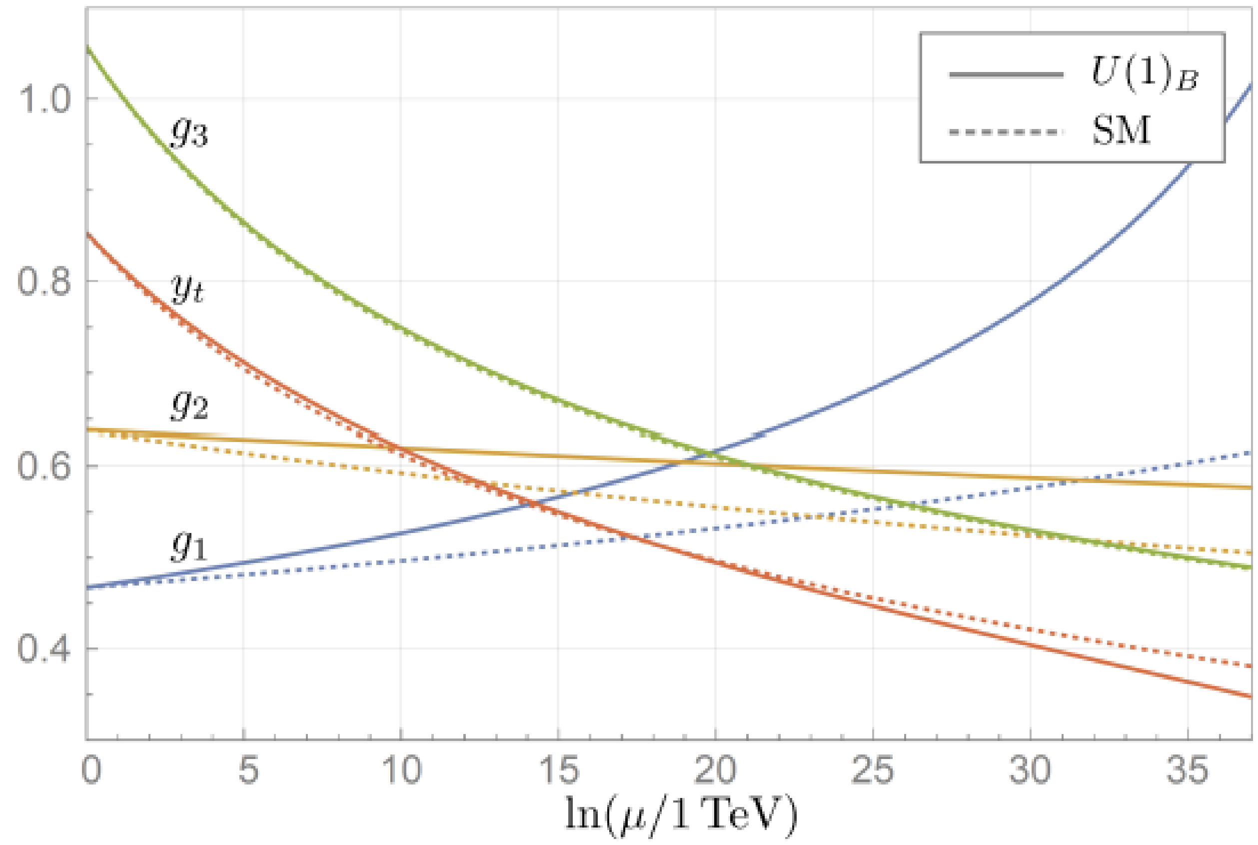




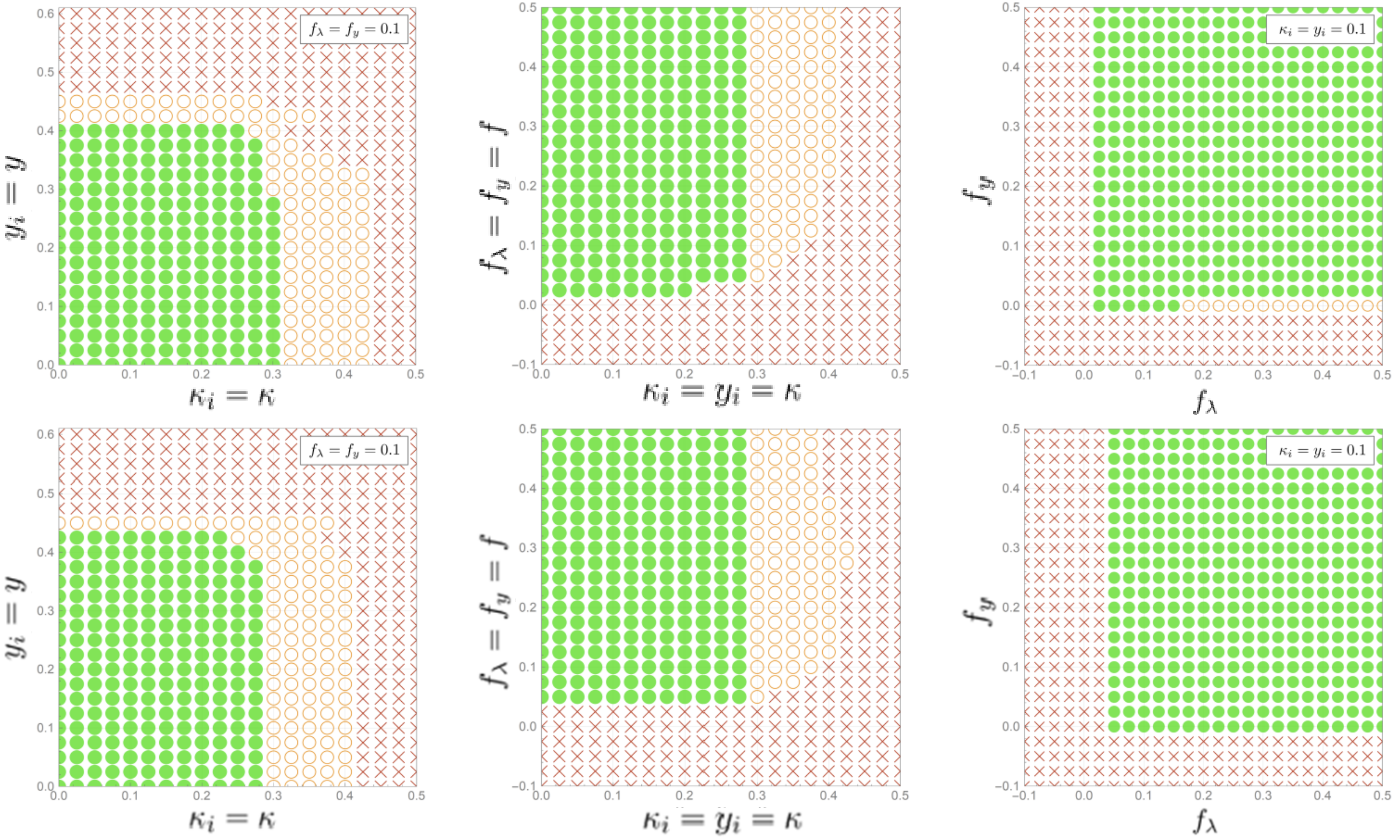
Interacting g_1 fixed point

$$\lambda_\star = 0.018$$
$$\lambda_{\phi\star} = 0.32$$
$$\lambda_{m\star} = 0.0037$$

Comparison with the Standard Model



Exploring the Parameter Space



Gaussian Fixed Point

Interacting Fixed Point

- Parameter choices at $\mu_0 = 1$ TeV
- ● = Asymptotically safe
- ○ = Metastable vacuum
- × = Not asymptotically safe

Partial Decay Widths for the Z' Boson

- For example, consider the interacting g_1 fixed point:
  - $\tilde{g} = \frac{16}{77} g_B$ for $0 \leq g_B \leq 0.4013$
- Partial decay widths:

$$\Gamma(f\bar{f}) = \frac{N_c}{48\pi} \left(C_V^2 + C_A^2\right) m_B g_B^2$$

$$\Gamma(W^+W^-) = \frac{\alpha \cos\theta_w^2 \epsilon^2}{12} m_B \frac{y^4}{(1-y^2)^2} \sqrt{1 - \frac{1}{x^2}} \left(4x^4 + 16x^2 - \frac{3}{x^2} - 17\right)$$

- $x = \frac{m_B}{2m_W}$, $y = \frac{m_Z}{m_B}$, θ_W is the weak mixing angle
- For $m_B \gg m_Z$ and small $\tilde{g}$: $\Gamma(W^+W^-) \approx \frac{4}{29645\pi} m_B g_B^2$

Branching Fractions for the Z' Boson

- The choice $m_B = 3$ TeV and $g_B = 0.3$ (0.6 in plot conventions) is consistent with assumptions

$$\mathrm{BF}(Z' \to \mathrm{jets}) = 77.8\,\%$$

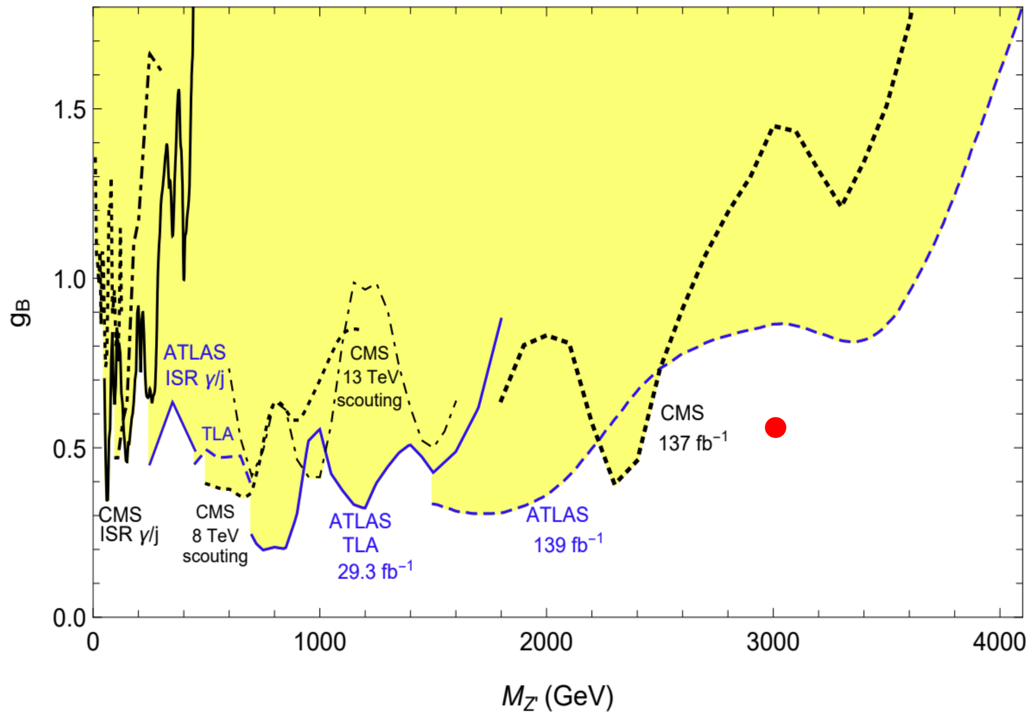
$$\mathrm{BF}(Z' \to t\bar{t}) = 19.8\,\%$$

$$\mathrm{BF}(Z' \to \ell^+\ell^-) = 2.0\,\%$$

$$\mathrm{BF}(Z' \to W^+W^-) = 0.1\,\%$$

- $g_B = 0.3 \to \tilde{g} = 0.062 \to \epsilon = 0.13$
- For Z' of order the TeV scale, the kinetic mixing bound is $\epsilon \lesssim 0.3$

(A. Hook, E. Izaguirre, J.G. Wacker, arXiv:1006.0973)



(B.A. Dobrescu and F. Yu, arXiv:2112.05392)

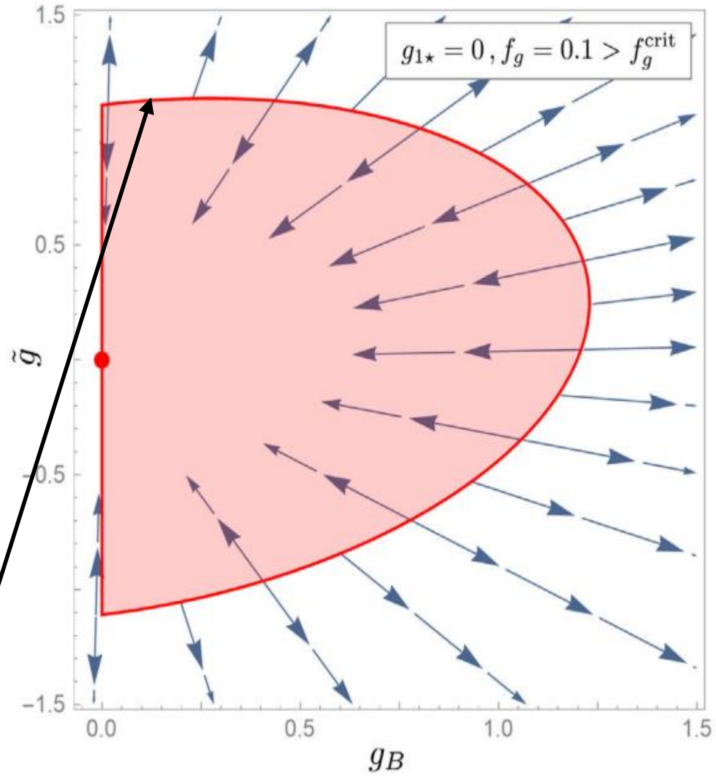
Talk Outline

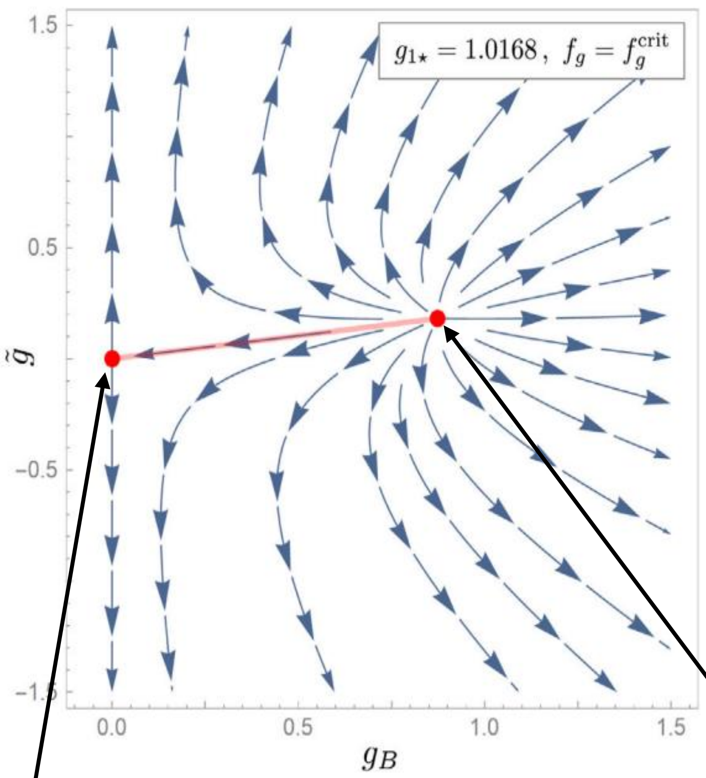
- Background: Renormalization group equations and asymptotic safety
- The gauged baryon number model and its various sectors
- The UV fixed point structure
- Numerical analysis and phenomenology
- Dark matter analysis
    - Dark matter stability
    - Relic density
    - Direct detection

Dark Matter Stability

- All particles (except χ), have baryon number charge $|Q_B| = 0, \frac{1}{3}, \text{or } 1$
- χ has $Q_B = 1/6$
- Under a $U(1)_B$ phase rotation: $e^{iQ_B(6\pi)}$, all fields are left invariant except for $\chi \rightarrow -\chi$
- This $\mathbb{Z}_2 \subset U(1)_B$ remains unbroken after spontaneous symmetry breaking. It is safe from violation by quantum gravitational effects
- Since χ is the only field odd under this symmetry, its stability is guaranteed

Ultraviolet Fixed Points





Benchmark Fixed Point: (0.202, 1.136)
At 1 TeV: (0.200, 0.150)

(ii)

Fixed Point: (0,0)
At 1 TeV: (0.200,0.041)

(ib)

Fixed Point: (0.871, 0.181)
At 1 TeV: (0.400, 0.083)

(ia)

Dark Matter Relic Density

- Thermal equilibrium exists if DM annihilation rate to SM particles exceeds the expansion rate. $\sigma(\chi\bar{\chi} \to f\bar{f})$ via Z' exchange dominates:

$$\sigma\left(\chi\bar{\chi} \to f\bar{f}\right) = \frac{N_c\, g_B^4}{1728\,\pi}\frac{1}{s}\sqrt{\frac{s-4\,m_f^2}{s-4\,m_\chi^2}}\,(s+2\,m_\chi^2)\left[\frac{C_V^2(s+2\,m_f^2)+C_A^2(s-4\,m_f^2)}{(s-m_B^2)^2+\Gamma^2\,m_B^2}\right]$$

- The decay width of Z' to standard model fermions is:

$$\Delta\Gamma\left(\,Z' \to f\bar{f}\right) = \frac{N_c\, g_B^2\, m_B}{48\,\pi}\sqrt{1-\frac{4m_f^2}{m_B^2}}\left[C_V^2\left(1+\frac{2m_f^2}{m_B^2}\right)+C_A^2\left(1-\frac{4m_f^2}{m_B^2}\right)\right]$$

- We use a relativistic treatment of the thermally averaged annihilation cross section times relative velocity:

$$\langle\sigma v\rangle = \frac{1}{8m_\chi^4 T K_2^2\left(\frac{m_\chi}{T}\right)}\int_{4m_\chi^2}^{\infty} \mathrm{d}s\,\sigma_{tot}\times(s-4m_\chi^2)\sqrt{s}K_1\left(\frac{\sqrt{s}}{T}\right)$$

Dark Matter Relic Density

- χ falls out of thermal equilibrium at the freeze-out temperature:

Freeze-out condition $\longrightarrow$ $$\frac{n_\chi^{EQ}\langle\sigma v\rangle}{H(T)} \approx 1 \qquad n_\chi^{\mathrm{EQ}} = 2\left(\frac{m_\chi T}{2\pi}\right)^{3/2} e^{-m_\chi/T} \qquad H(T) = 1.66\sqrt{g_*}\,T^2/M_{\mathrm{Pl}}$$

- The entropy density to number density ratio is then propagated from the freeze-out temperature to the present temperature:

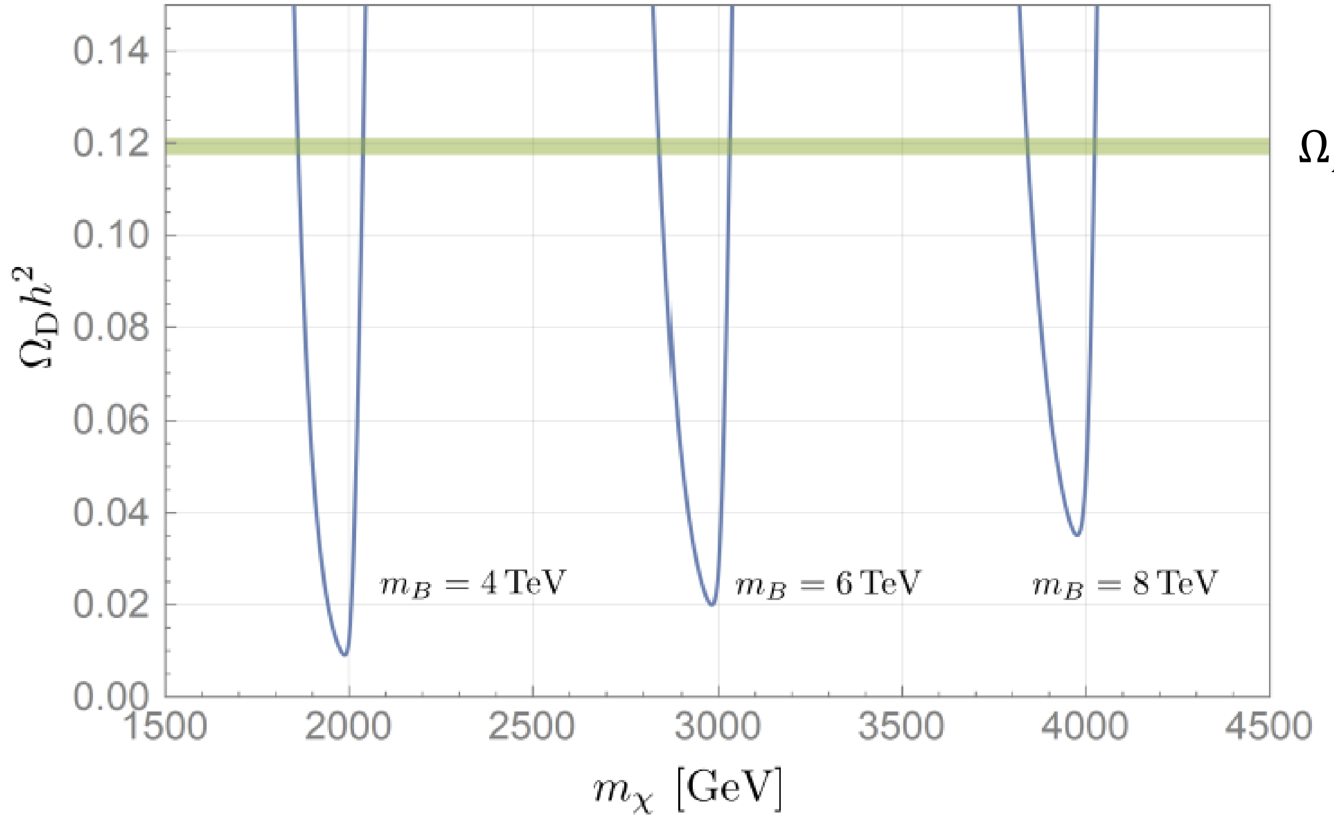
$$Y_f = 0.145\frac{g}{g_*}x_f^{3/2}e^{-x_f} \qquad \frac{1}{Y_0} = \frac{1}{Y_f} + \sqrt{\frac{\pi}{45}}M_{\mathrm{Pl}}m_\chi \int_{x_f}^{x_0} \mathrm{d}x \frac{\sqrt{g_*}}{x^2}\frac{\langle\sigma v\rangle}{2} \qquad x_f \equiv m_\chi/T_f$$

- The dark matter relic density is then:

$$\Omega_{\mathrm{D}}h^2 \approx \frac{2.8\times 10^8}{\mathrm{GeV}} Y_0\, m_\chi$$

Dark Matter Relic Density

- We illustrate the relic density results for different TeV scale m_χ and m_B in the case of $(g_1, g_B, \tilde{g})$ all flowing to the interacting fixed point:

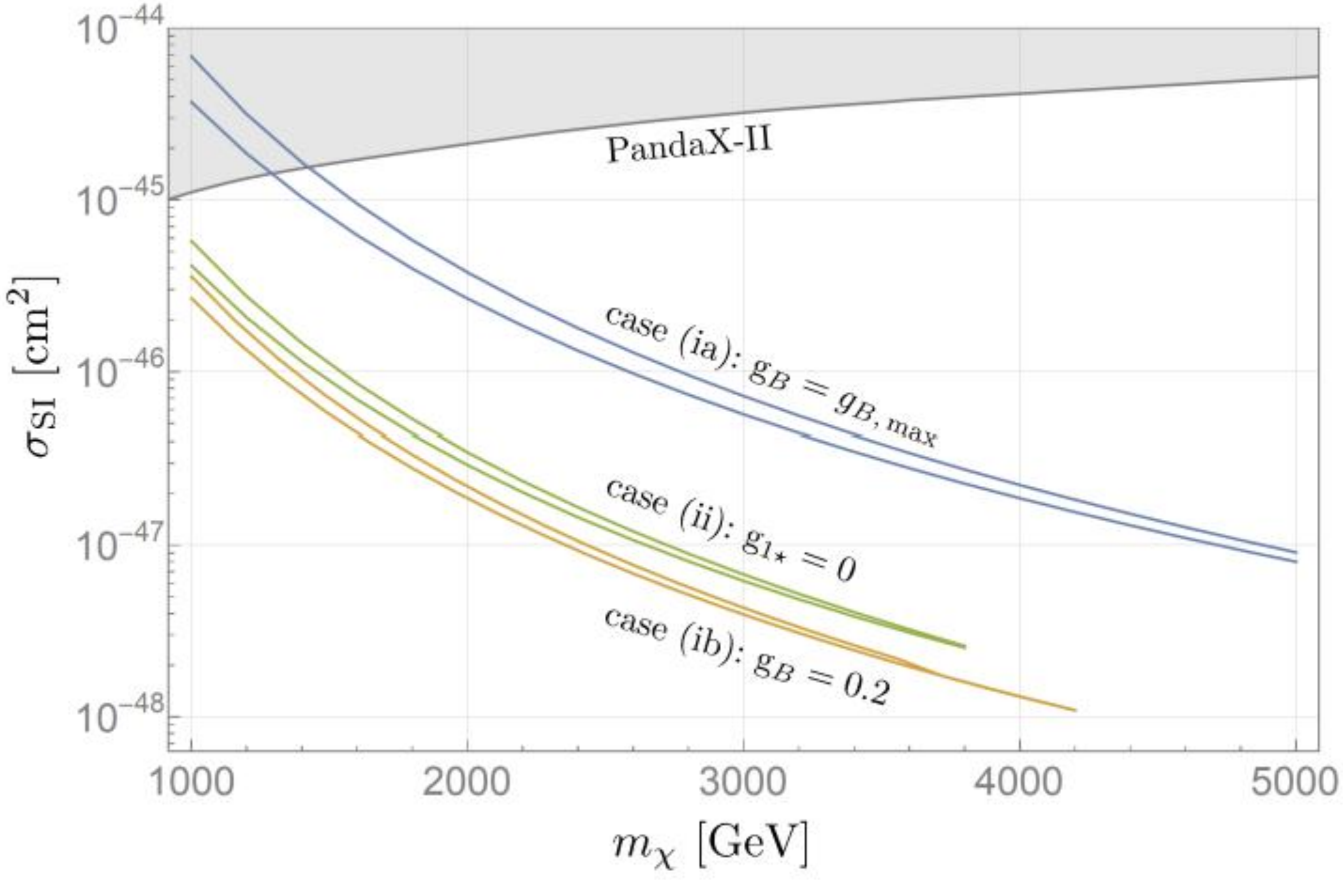


$\Omega_D h^2 = 0.1193 \pm 0.0009$
(Planck measurement)

Direct Detection

- For each m_χ, we pick an m_B which achieves the correct relic density

$$\sigma_{\rm SI} = \frac{g_B^4}{36\,\pi}\frac{\mu_{\chi N}^2}{m_B^4}\left(1 + 0.35\,\frac{\tilde{g}}{g_B}\right)^2$$



Conclusions

1. Gravitational corrections above the Planck scale allow for different fixed-point scenarios
2. Requiring asymptotic safety constrains the model's couplings, including the kinetic mixing parameter, and yields a more predictive theory
3. The model includes a stable TeV-scale dark matter candidate
4. These models can predict the observed relic density while also remaining consistent with direct detection experiments

Thank You!

RGEs of Gauge Couplings at 1 Loop

$$\beta^{(1)}(g_1) = +\frac{77}{10} g_1^3 \,,$$

$$\beta^{(1)}(g_B) = +11 g_B^3 + \frac{77}{6} g_B \tilde{g}^2 - \frac{16}{3} g_B^2 \tilde{g} \,,$$

$$\beta^{(1)}(\tilde{g}) = -\frac{16}{5} g_1^2 g_B - \frac{16}{3} g_B \tilde{g}^2 + \frac{77}{5} g_1^2 \tilde{g} + 11 g_B^2 \tilde{g} + \frac{77}{6} \tilde{g}^3 \,,$$

$$\beta^{(1)}(g_2) = -\frac{7}{6} g_2^3 \,,$$

$$\beta^{(1)}(g_3) = -7 g_3^3 \,.$$

$$\beta(c) = \sum_{\ell=1}^{\infty} \frac{1}{(4\pi)^{2\ell}} \beta^{(\ell)}(c)$$

RGEs of Yukawa Couplings at 1 Loop

Yukawas ($\kappa^2 \equiv \kappa_1^2 + \kappa_2^2 + \kappa_3^2$, $y^2 \equiv 2y_1^2 + y_2^2 + y_3^2$):

$$\beta^{(1)}(y_t) = \left(\frac{9}{2}y_t^2 + \frac{3}{2}y_b^2 + 3\kappa^2 - \frac{17}{20}g_1^2 - \frac{9}{4}g_2^2 - 8g_3^2 - \frac{2}{3}g_B^2 - \frac{5}{3}g_B\tilde{g} - \frac{17}{12}\tilde{g}^2\right) y_t\,,$$

$$\beta^{(1)}(y_b) = \left(\frac{3}{2}y_t^2 + \frac{9}{2}y_b^2 + 3\kappa^2 - \frac{1}{4}g_1^2 - \frac{9}{4}g_2^2 - 8g_3^2 - \frac{2}{3}g_B^2 + \frac{1}{3}g_B\tilde{g} - \frac{5}{12}\tilde{g}^2\right) y_b\,,$$

$$\beta^{(1)}(y_1) = \left(3y^2 + y_1^2 + \frac{1}{2}\kappa_1^2 - \frac{9}{10}g_1^2 - \frac{9}{2}g_2^2 - 3g_B^2 + 3g_B\tilde{g} - \frac{3}{2}\tilde{g}^2\right) y_1\,,$$

$$\beta^{(1)}(y_2) = \left(3y^2 + y_2^2 + \kappa_2^2 - \frac{18}{5}g_1^2 - 3g_B^2 + 6g_B\tilde{g} - 6\tilde{g}^2\right) y_2\,,$$

$$\beta^{(1)}(y_3) = \left(3y^2 + y_3^2 + \kappa_3^2 - 3g_B^2\right) y_3\,,$$

$$\beta^{(1)}(\kappa_1) = \left(3y_t^2 + 3y_b^2 + \frac{1}{2}y_1^2 + \frac{9}{2}\kappa_1^2 + 3\kappa_2^2 + 3\kappa_3^2 - \frac{9}{4}g_1^2 - \frac{9}{4}g_2^2 - \frac{15}{4}\tilde{g}^2\right) \kappa_1\,,$$

$$\beta^{(1)}(\kappa_2) = \left(3y_t^2 + 3y_b^2 + \frac{1}{2}y_2^2 + 3\kappa_1^2 + \frac{9}{2}\kappa_2^2 + \frac{3}{2}\kappa_3^2 - \frac{9}{4}g_1^2 - \frac{9}{4}g_2^2 - \frac{15}{4}\tilde{g}^2\right) \kappa_2\,,$$

$$\beta^{(1)}(\kappa_3) = \left(3y_t^2 + 3y_b^2 + \frac{1}{2}y_3^2 + 3\kappa_1^2 + \frac{3}{2}\kappa_2^2 + \frac{9}{2}\kappa_3^2 - \frac{9}{20}g_1^2 - \frac{9}{4}g_2^2 - \frac{3}{4}\tilde{g}^2\right) \kappa_3\,.$$

$$\beta(c) = \sum_{\ell=1}^{\infty} \frac{1}{(4\pi)^{2\ell}}\beta^{(\ell)}(c)$$

RGEs of Quartic Couplings at 1 Loop

Quartic couplings ($\kappa^2 \equiv \kappa_1^2+\kappa_2^2+\kappa_3^2$, $K^4 \equiv \kappa_1^4+\kappa_2^4+\kappa_3^4$, $y^2 \equiv 2y_1^2+y_2^2+y_3^2$, $Y^4 \equiv 2y_1^4+y_2^4+y_3^4$):

$$\begin{aligned}
\beta^{(1)}(\lambda) &= +12\lambda^2 + 2\lambda_m^2 - \frac{9}{5}g_1^2\lambda - 9g_2^2\lambda - 3\tilde{g}^2\lambda \\
&+ \frac{27}{100}g_1^4 + \frac{9}{10}g_1^2 g_2^2 + \frac{9}{10}g_1^2\tilde{g}^2 + \frac{9}{4}g_2^4 + \frac{3}{2}g_2^2\tilde{g}^2 + \frac{3}{4}\tilde{g}^4 \\
&+ 12\lambda\left(y_t^2 + y_b^2 + \kappa^2\right) - 12\left(y_t^4 + y_b^4 + K^4\right) , \\
\beta^{(1)}(\lambda_\phi) &= +10\lambda_\phi^2 + 4\lambda_m^2 - 12g_B^2\lambda_\phi + 12g_B^4 + 12\lambda_\phi y^2 - 12Y^4 , \\
\beta^{(1)}(\lambda_m) &= \left[6\lambda + 4\lambda_\phi + 4\lambda_m - \frac{9}{10}g_1^2 - \frac{9}{2}g_2^2 - 6g_B^2 - \frac{3}{2}\tilde{g}^2 + 6(y_t^2 + y_b^2 + y^2 + \kappa^2)\right]\lambda_m \\
&+ 3g_B^2\tilde{g}^2 - 12(\kappa_1^2 y_1^2 + \kappa_2^2 y_2^2 + \kappa_3^2 y_3^2) .
\end{aligned}$$

$$\beta(c) = \sum_{\ell=1}^{\infty} \frac{1}{(4\pi)^{2\ell}} \beta^{(\ell)}(c)$$

1 TeV SM Coupling Values

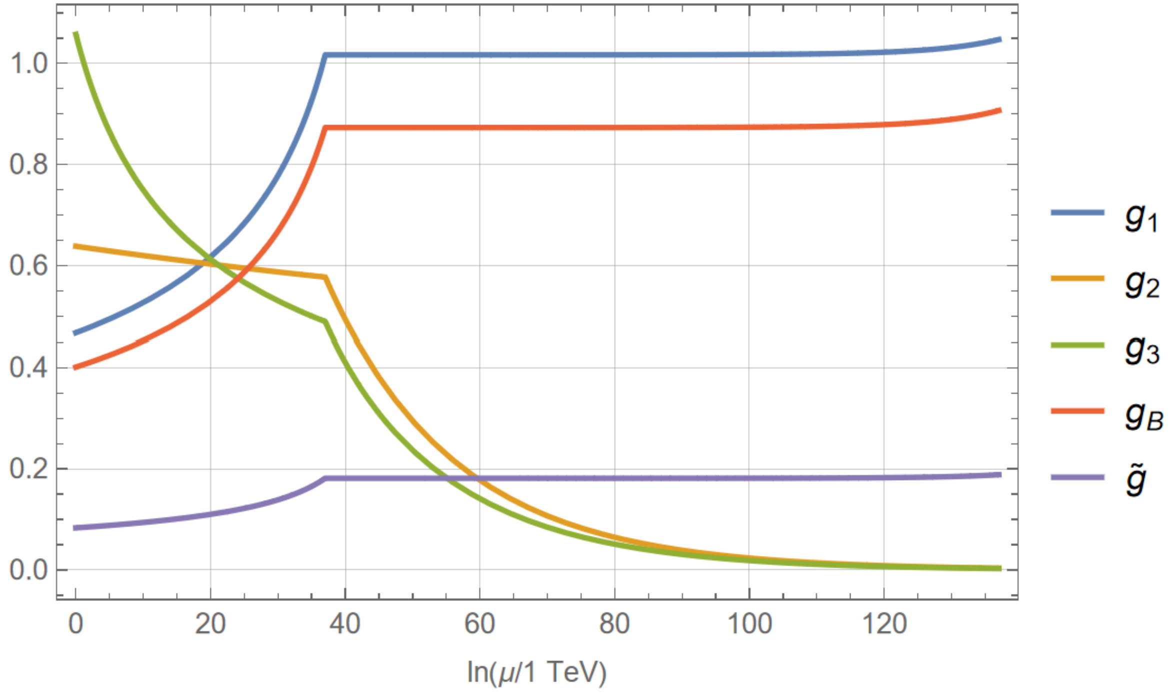
- SM Higgs mass is 125 GeV and the Higgs vev is 246 GeV. The following are the measured gauge coupling values at $\mu_0 = 1$ TeV and the Yukawa couplings for the top and bottom quarks:

$$g_1(\mu_0) = 0.46738, \qquad g_2(\mu_0) = 0.63829, \qquad g_3(\mu_0) = 1.05737,$$

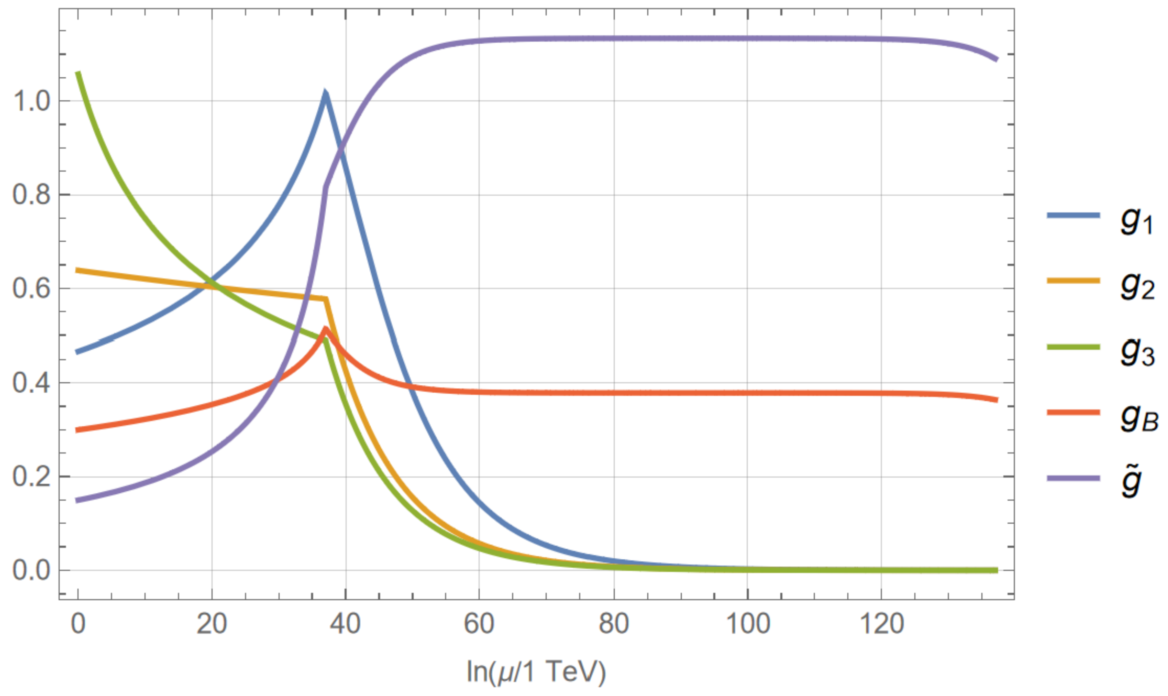
$$y_t(\mu_0) = 0.85322, \qquad y_b(\mu_0) = 0.01388\,.$$

(G. Hiller, C. Hormigos-Feliu, D. F. Litim and T. Steudtner, arXiv:2008.08606)

Evolution of Gauge Couplings



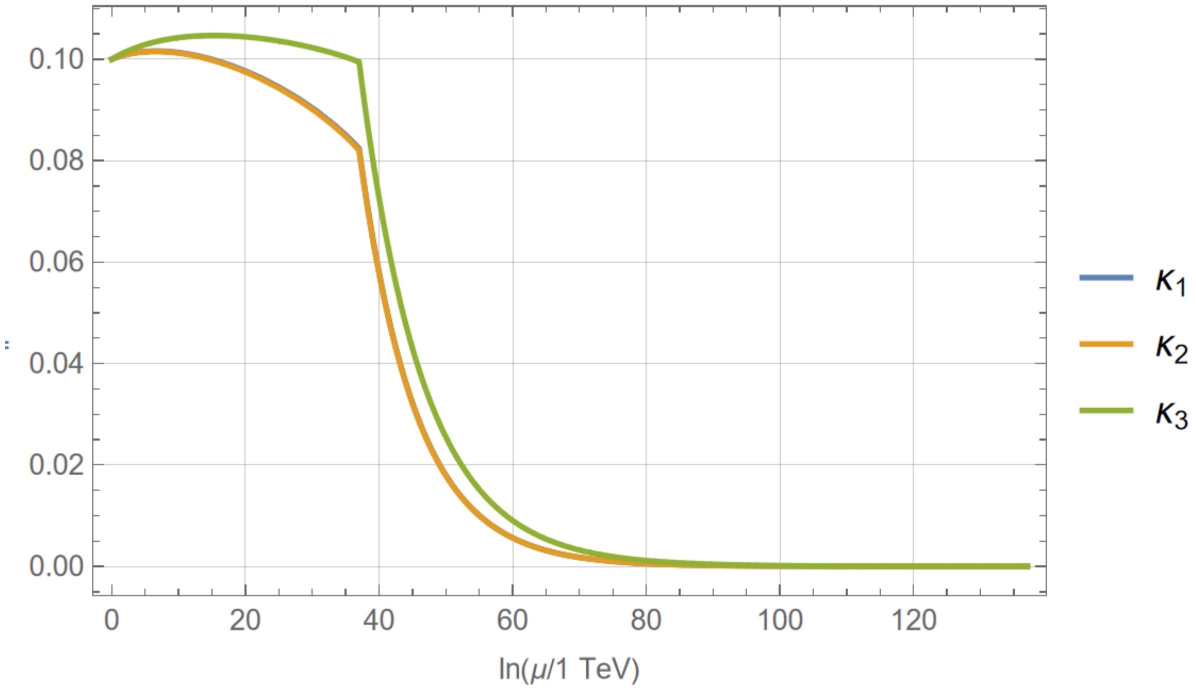
Interacting g_1 fixed point



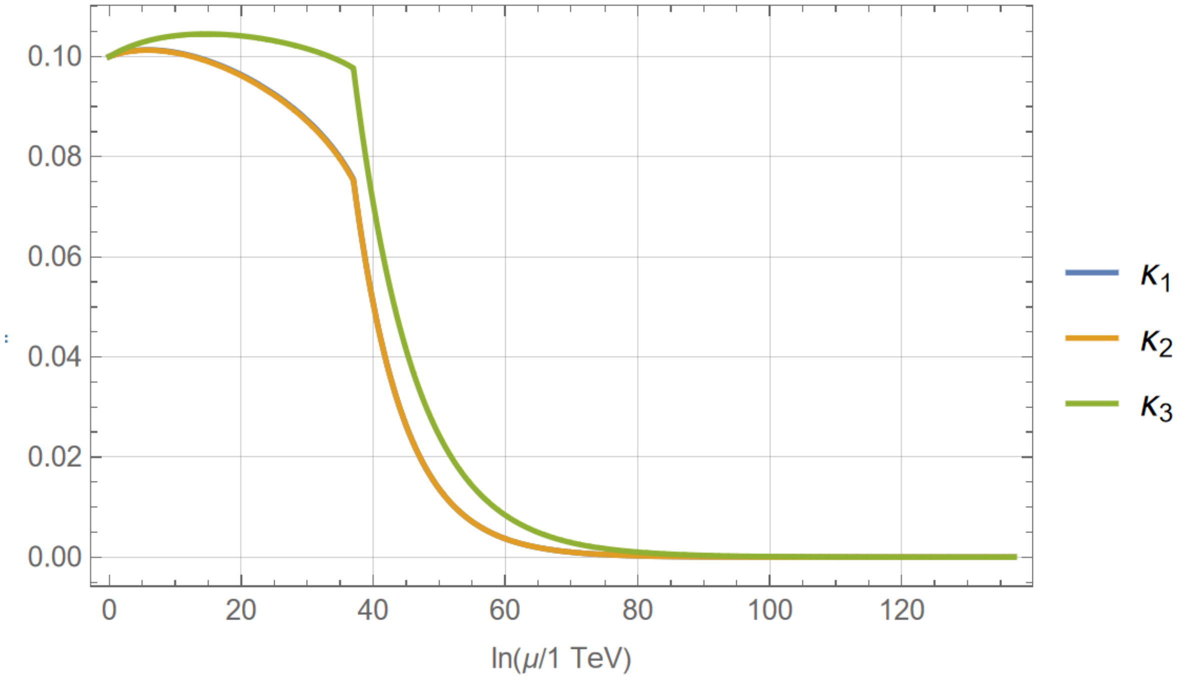
Gaussian g_1 fixed point

Evolution of Yukawa Couplings

$$\mathcal{L}_\kappa = \overline{\psi^\ell_L}\, H\, \kappa_1 e_R + \overline{\ell_L}\, H\, \kappa_2\, \psi^e_R + \overline{\ell_L}\, \widetilde{H}\, \kappa_3 \psi^\nu_R + \text{ H.c.}$$



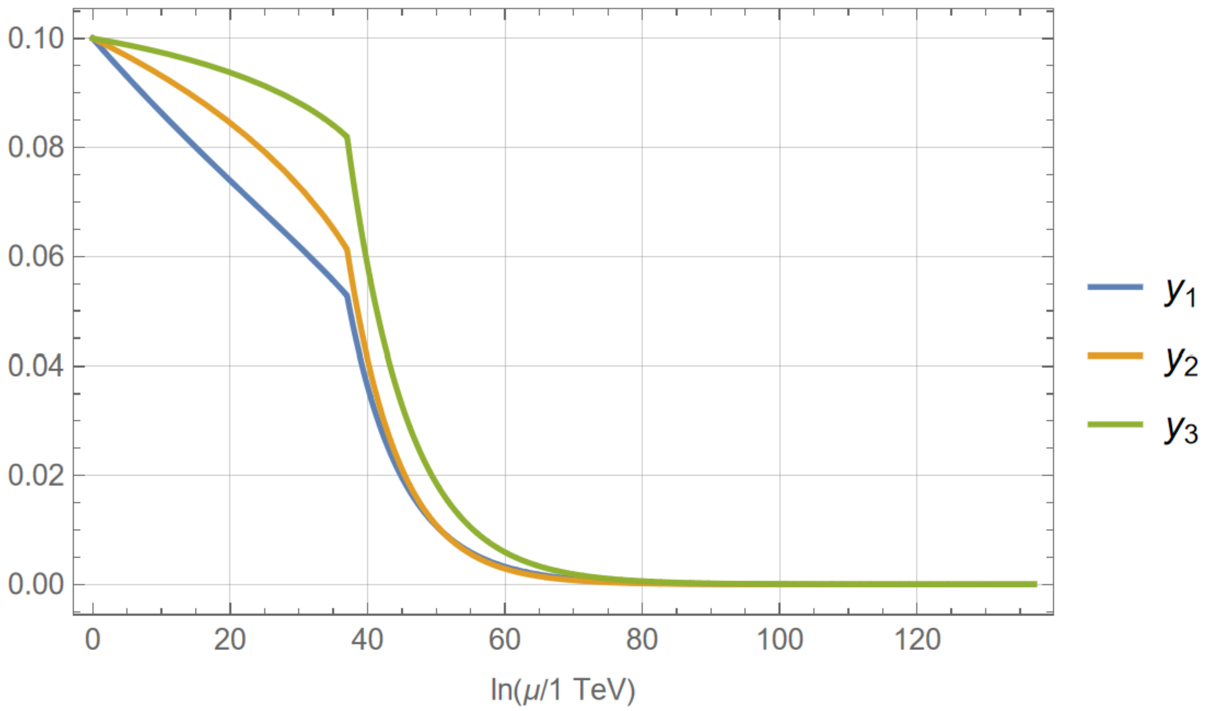
Interacting g_1 fixed point



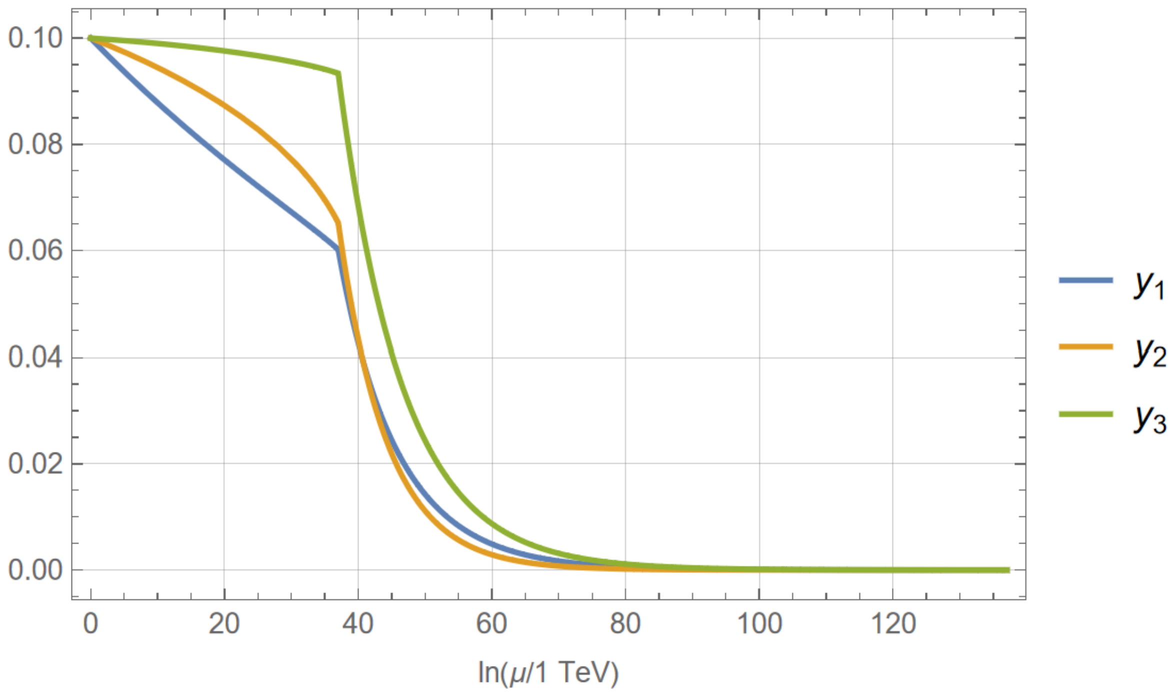
Gaussian g_1 fixed point

Evolution of Yukawa Couplings

$$\mathcal{L}_y = \overline{\psi_L^\ell}\, y_1\, \psi_R^\ell\, \phi^* + \overline{\psi_L^e}\, y_2\, \psi_R^e\, \phi + \overline{\psi_L^\nu}\, y_3\, \psi_R^\nu\, \phi + \text{ H.c.}$$

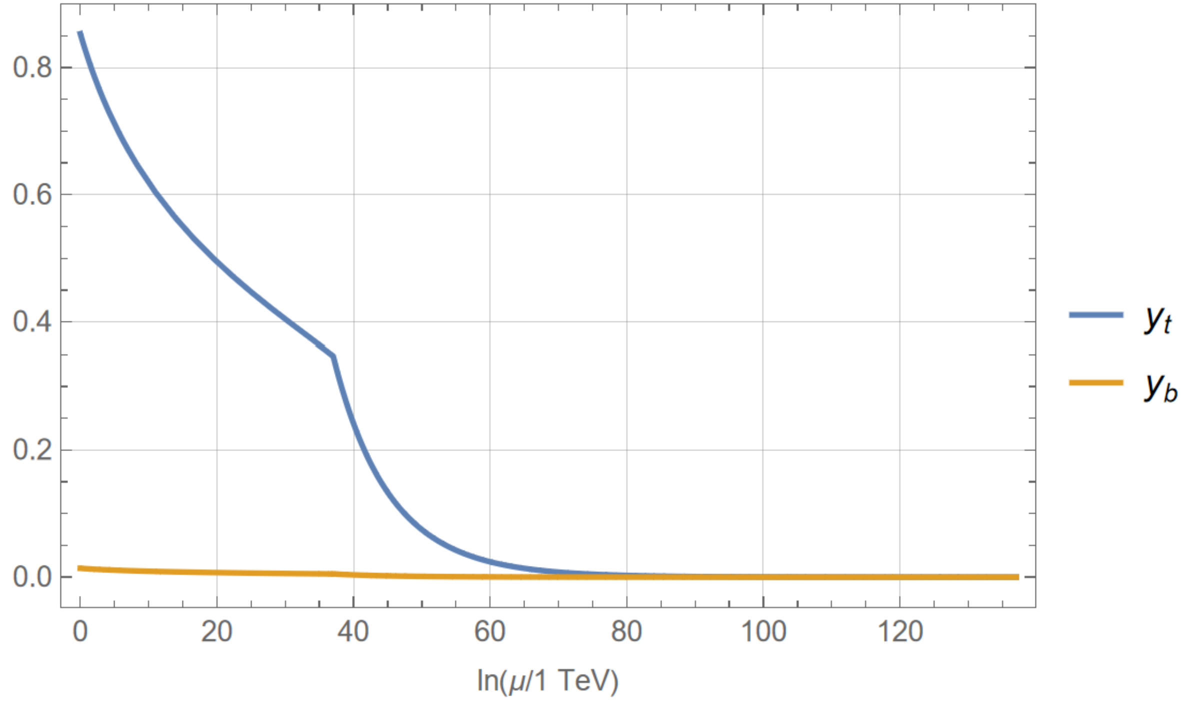


Interacting g_1 fixed point

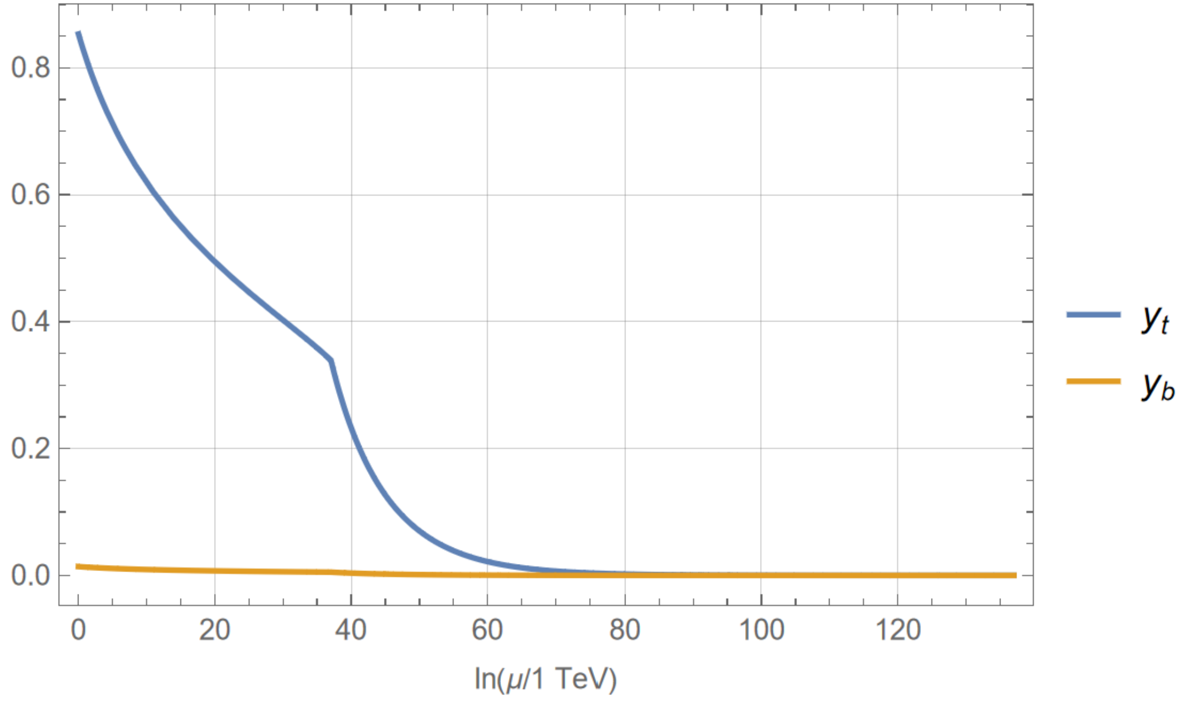


Gaussian g_1 fixed point

Evolution of Yukawa Couplings



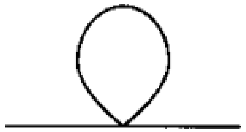
Interacting g_1 fixed point



Interacting g_1 fixed point

ϕ^4 Theory One Loop Self Energy

- Consider the self-energy at one loop:

$-i\Sigma(p^2) =$  $+$ 

- Apply dimensional regularization and $\overline{MS}$ renormalization scheme:

$$= -i\lambda\mu^{\epsilon}\frac{1}{2}\int\frac{d^dk}{(2\pi)^d}\frac{i}{k^2-m^2} + i\left(p^2\delta_Z - \delta_m\right) = -\frac{i\lambda\mu^{\epsilon}}{2}\frac{1}{(4\pi)^{\frac{d}{2}}}\frac{\Gamma\left(1-\frac{d}{2}\right)}{(m^2)^{1-\frac{d}{2}}} + i\left(p^2\delta_Z - \delta_m\right)$$

$$= \frac{i\lambda}{2}\frac{m^2}{(4\pi)^2}\left[\frac{2}{\epsilon} - \gamma + \ln(4\pi) + 1 - \ln\left(\frac{m^2}{\mu^2}\right)\right] + i\left(p^2\delta_Z - \delta_m\right)$$

- So, $\delta_Z = 0$ and $\delta_m = \frac{\lambda m^2}{32\pi^2}\left[\frac{2}{\epsilon} - \gamma + \ln(4\pi)\right] = m_0^2 Z - m^2 = m_0^2 - m^2$ (at one loop)

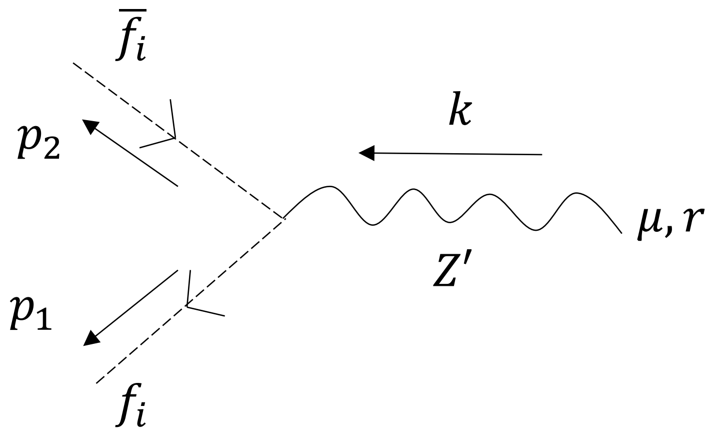
Identifying C_V and C_A

Consider the expansion of the kinetic term: $i\left(\overline{P_L\psi}\,\gamma^\mu D_\mu^L\,P_L\psi + \overline{P_R\psi}\,\gamma^\mu D_\mu^R\,P_R\psi\right)$

- $D_\mu^{L,R} = \left(\partial_\mu - ig_B Q_B^{L,R} B_\mu - i\left(g_Y A_\mu^Y + \tilde{g} B_\mu\right) Q_Y^{L,R}\right)$
- $P_L = \frac{1-\gamma^5}{2}$, $P_R = \frac{1+\gamma^5}{2}$, also note that $\left(\gamma^5\right)^2 = 1$
- $\overline{P_L\psi} = \bar{\psi} P_R$ and likewise $\overline{P_R\psi} = \bar{\psi} P_L$
- Since we are interested in the coupling to Z' we will just focus on these terms in the covariant derivative:

- $i\,\frac{1+\gamma^5}{2}\left(-ig_B Q_B^L \gamma^\mu - i\tilde{g} Q_Y^L \gamma^\mu\right)\frac{1-\gamma^5}{2} + i\,\frac{1-\gamma^5}{2}\left(-ig_B Q_B^R \gamma^\mu - i\tilde{g} Q_Y^R \gamma^\mu\right)\frac{1+\gamma^5}{2}$
- $\frac{1}{2}\left[\left(g_B Q_B^L \gamma^\mu + \tilde{g} Q_Y^L \gamma^\mu - 2g_B Q_B^L \gamma^\mu \gamma^5 - \tilde{g} Q_Y^L \gamma^\mu \gamma^5\right) + \left(g_B Q_B^R \gamma^\mu + \tilde{g} Q_Y^R \gamma^\mu + g_B Q_B^R \gamma^\mu \gamma^5 + \tilde{g} Q_Y^R \gamma^\mu \gamma^5\right)\right]$
- $\frac{1}{2}\gamma^\mu\left[g_B\left(Q_B^L + Q_B^R\right) + \tilde{g}\left(Q_Y^L + Q_Y^R\right) + g_B\left(Q_B^R - Q_B^L\right)\gamma^5 + \tilde{g}\left(Q_Y^R - Q_Y^L\right)\gamma^5\right]$
- $C_V = g_B\left(Q_B^L + Q_B^R\right) + \tilde{g}\left(Q_Y^L + Q_Y^R\right)$
- $C_A = g_B\left(Q_B^R - Q_B^L\right)\gamma^5 + \tilde{g}\left(Q_Y^R - Q_Y^L\right)$

Partial Decay Widths



$k = p_1 + p_2$

$$i\mathcal{M}^j = \bar{u}^{s_1}(p_1)\frac{i}{2}\left[C_V^j\gamma^\mu + C_A^j\gamma^\mu\gamma^5\right]v^{s_2}(p_2)\varepsilon_\mu^r(k)$$

$$|\mathcal{M}^j|^2 = \frac{1}{3}\Sigma_r\Sigma_{s_1,s_2}\mathcal{M}^{j*}\mathcal{M}^j$$

$\Gamma^j \propto \left|\mathcal{M}^j\right|^2$ and $\Gamma = \Sigma_j\Gamma^j$

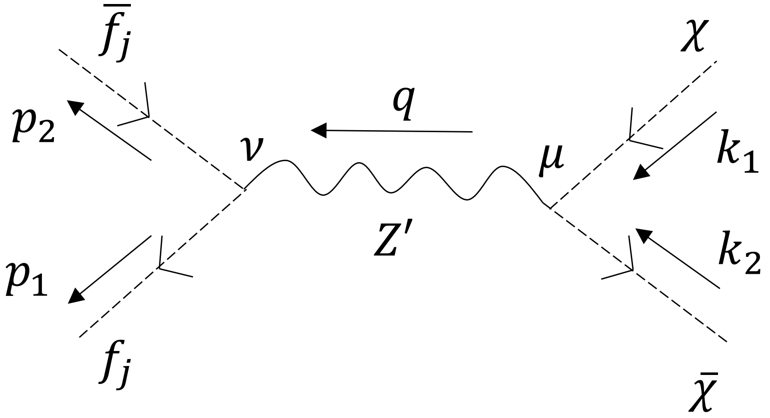
Values for interacting g_1 fixed point

Species	$\|C_V\|/g_B$	$\|C_A\|/g_B$
up-type quark	0.8008	0.0805
down-type quarks	0.6398	0.0805
charged leptons	0.2414	0.0805
neutrinos	0.0805	0.0805

$$C_V = g_B\left(Q_B^L + Q_B^R\right) + \tilde{g}\left(Q_Y^L + Q_Y^R\right)$$

$$C_A = g_B\left(Q_B^R - Q_B^L\right) + \tilde{g}\left(Q_Y^R - Q_Y^L\right)$$

DM Annihilation to SM Fermions



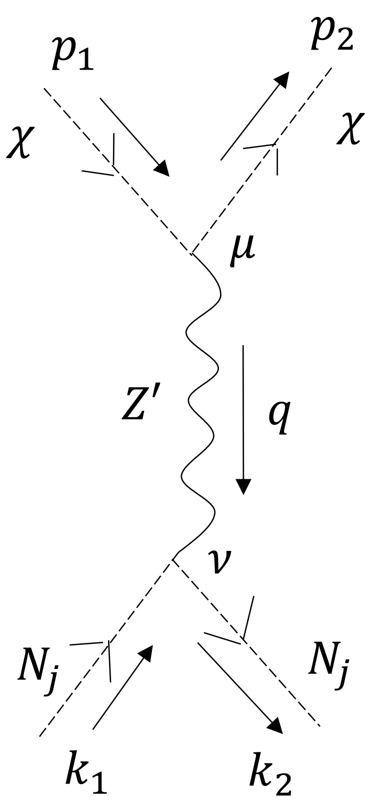
$$k_1 + k_2 = q = p_1 + p_2$$

$$i\mathcal{M}^j = \bar{v}^{s_2}(k_2)\left(\frac{ig_B}{3}\gamma^\mu\right)u^{s_2}(k_1)\left[-\frac{i\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{m_B^2}\right)}{q^2 - m_B^2 + im_B\Gamma}\right]\bar{u}^{r_1}(p_1)\left(\frac{i}{2}\left(C_V^j\gamma^\nu + C_A^j\gamma^\nu\gamma^5\right)\right)v^{r_2}(p_2)$$

$$\left|\mathcal{M}^j\right|^2 = \frac{1}{4}\Sigma_{s_1,s_2}\Sigma_{r_1,r_2}\mathcal{M}^{j*}\mathcal{M}^j$$

$\sigma^j \propto \left|\mathcal{M}^j\right|^2$ and $\sigma = \Sigma_j \sigma^j$

Direct Detection Calculation



$$p_1 - p_1 = q = k_2 - k_1$$

Consider scattering off of a Xenon-132 atom:

$$i\mathcal{M}^j = \bar{u}^{s_2}(p_2)\left(\frac{i}{2}\right)\left(\frac{g_B}{3}\gamma^\mu\right)u^{s_1}(p_1)\left[-\frac{i\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{m_B^2}\right)}{q^2 - m_B^2}\right]\bar{u}^{r_2}(k_2)\left(\frac{i}{2}\left(C_V^j\gamma^\nu + C_A^j\gamma^\nu\gamma^5\right)\right)u^{r_1}(k_1)$$

Xenon-132:
- $n_1 = 54$ protons
- $n_2 = 78$ neutrons
- $m_N \approx 939\ MeV$

For protons, $C_V = 2g_B + \frac{3}{2}\tilde{g}$ and $C_A = -\frac{1}{2}\tilde{g}$

For neutrons, $C_V = 2g_B + \frac{1}{2}\tilde{g}$ and $C_A = \frac{1}{2}\tilde{g}$

- $\sigma_{eff} \propto \frac{\left|n_1\mathcal{M}^1 + n_2\mathcal{M}^2\right|^2}{(n_1+n_2)^2}$
- since the entire nucleus recoils, the individual protons/neutrons are not treated as distinct
- σ_{eff} is the effective spin-independent nucleon scattering cross section.