Towards a quantum field theory description of nonlocal spacetime defects

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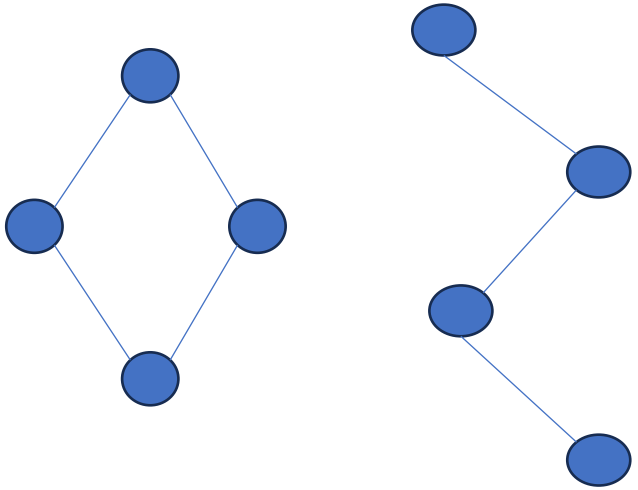
Causal Sets

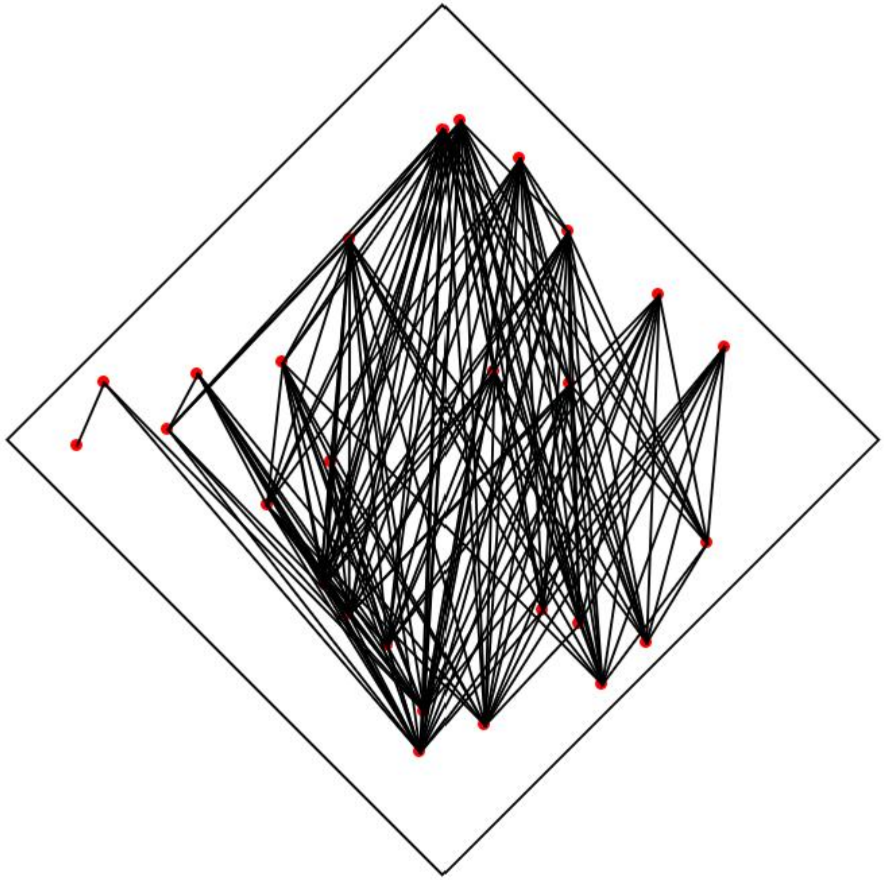
- Causal Set Theory (CST) is one approach towards a quantum theory of gravity
- A causal set is defined as a locally finite, partially ordered set $(C, \leq)$ satisfying the following relations for any $v_i, v_j, v_k \in C$:
  - Reflexive: $v_i \leq v_i$
  - Transitive: If $v_i \leq v_j$ and $v_j \leq v_k$, then $v_i \leq v_k$
  - Antisymmetric: If $v_i \leq v_j$ and $v_j \leq v_i$, then $v_i = v_j$
  - Locally finite: If $v_i \leq v_k$, then the set $\{v_l | v_i \leq v_l \leq v_k\}$ has finite cardinality
- Spacetime: $v_i \in C$ are events, $v_i \leq v_j$ if v_j is in the future light cone of v_i



Two inequivalent Hasse diagrams with four points

Causal Sets (cont.)

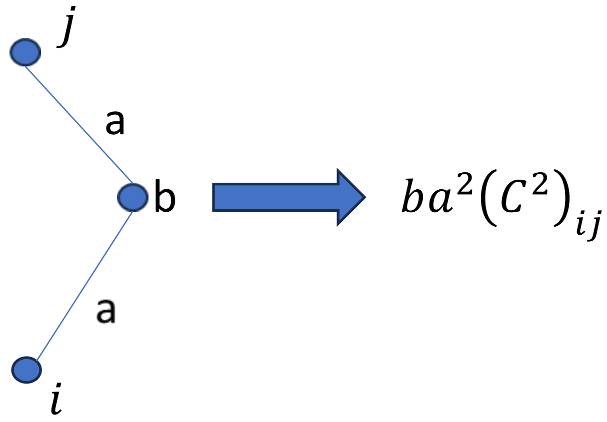
- A causal set can by described by a matrix called the causal matrix, C
  - $C_{ij} = 1$ if point v_j in the causal set is in the future light cone of point v_i. $C_{ij} = 0$ otherwise
- Poisson Sprinkling: To approximate a region of spacetime, Poisson distribute points at density ρ into that region and then connect points based on inherited causality
- Averaging over such approximations yields the continuum



The Retarded Propagator in CST

- Consider a free real scalar field of mass M in 1+1 Minkowski space
- To study particle propagation in CST consider paths between any two points in a Poisson sprinkled causal set with density ρ
- Assign amplitude a to each edge and amplitude b to each internal point in a path
- Total amplitude to travel between any two points is:

$$K = I + \sum_{i=1}^{i=\infty} b^{i-1} a^i C^i \; = I + aC(I - baC)^{-1}$$



The Retarded Propagator in CST (Cont.)

- Define $K_R = K - I = aC(I - baC)^{-1}$
- By averaging over all Poisson sprinklings, it can be shown that:

$$< \widetilde{K_R} > (p) = -\frac{2a}{(p_0 + i\epsilon)^2 - p_1^2 + 2ab\rho}$$

- Set $a = \frac{1}{2}$ and $b = -\frac{M^2}{\rho}$

$$< \widetilde{K_R} > (p) = -\frac{1}{(p_0 + i\epsilon)^2 - p_1^2 - M^2}$$

The Feynman Propagator in CST

- In the continuum, we have a free real scalar quantum field $\phi(x)$ acting on a Hilbert space, H
- By restriction, let $\phi_x = \phi(x)$ be the causal set analog when x is in the causal set, C
- In the continuum theory, we have:

$$[\phi(x), \phi(y)] = i\Delta(x, y)$$

- Here, $\Delta(x, y) = K_R(x, y) - K_A(x, y)$ is the Pauli-Jordan function
- By analogy in CST, define the matrix $\Delta = K_R - K_R^T$
- Analogously we also have $\left[\phi_x, \phi_y\right] = (i\Delta)_{xy}$

The Feynman Propagator in CST (cont.)

- $i\Delta$ is Hermitian and skew-symmetric. So, $i\Delta$ has matching sets of positive and negative eigenvalues. Let $2s = rank(i\Delta)$ so that $i = 1, \dots, s$

$$(i\Delta)u_i = \lambda_i u_i \, , \, (i\Delta)v_i = -\lambda_i v_i$$

- The eigenvectors can be chosen orthonormal, $u_i = v_i^*$, and $u_i^\dagger v_j = 0$
- Define the following operators:

$$a_i^\dagger = \sum_{x \in C} (u_i)_x \phi_x \, , \, a_i = \sum_{x \in C} (v_i)_x \phi_x$$

The Feynman Propagator in CST (cont.)

- a_i and $a_i^\dagger$ satisfy canonical commutation relations. We can then express ϕ_x in terms of raising and lowering operators:

$$\phi_x = \sum_{i=1}^{i=s} (u_i)_x a_i + (v_i)_x a_i^\dagger$$

- Identify a normalized vacuum state $|0> \in H$ by $a_i|0> = 0$ for all i and $< 0|0> = 1$
- In the continuum, the Feynman propagator is defined as:

$$G_F(x,y) = i < 0|T\phi(x)\phi(y)|0 >$$

The Feynman Propagator in CST (cont.)

- By evaluating $(\mathrm{K_F})_{\mathrm{xy}} = i < 0\left|T\phi_x\phi_y\right|0 >$, it can be shown that:

$$K_F = K_R + iQ$$

- The matrix Q can be shown to be the causal set equivalent of the two-point function:

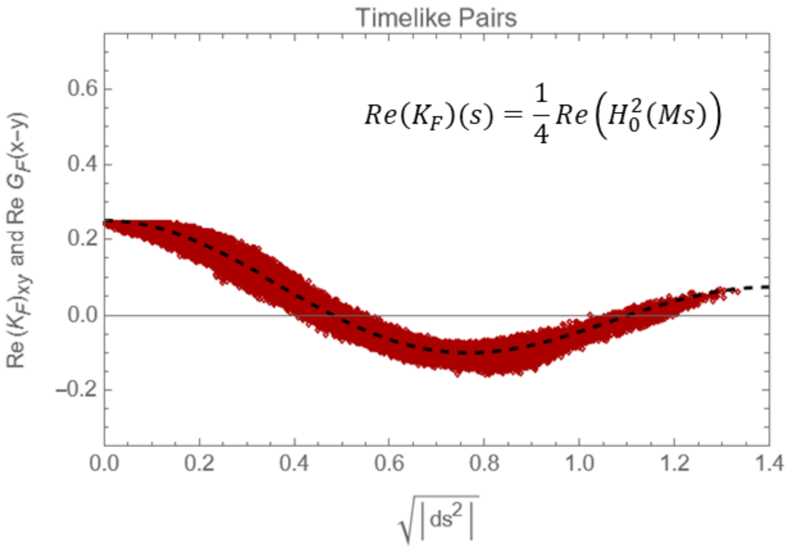
$$Q =< 0\left|\phi_x\phi_y\right|0 >= \sum_{\lambda_i>0} \lambda_i u_i u_i^\dagger$$

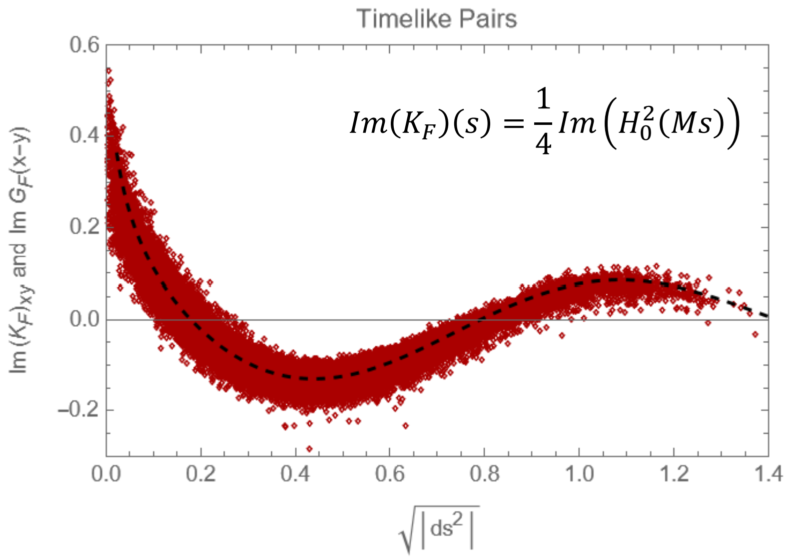
- $Re(K_F) = \frac{1}{2}\left(K_R + K_R^T\right)$ and $Im(K_F) = Re(Q)$

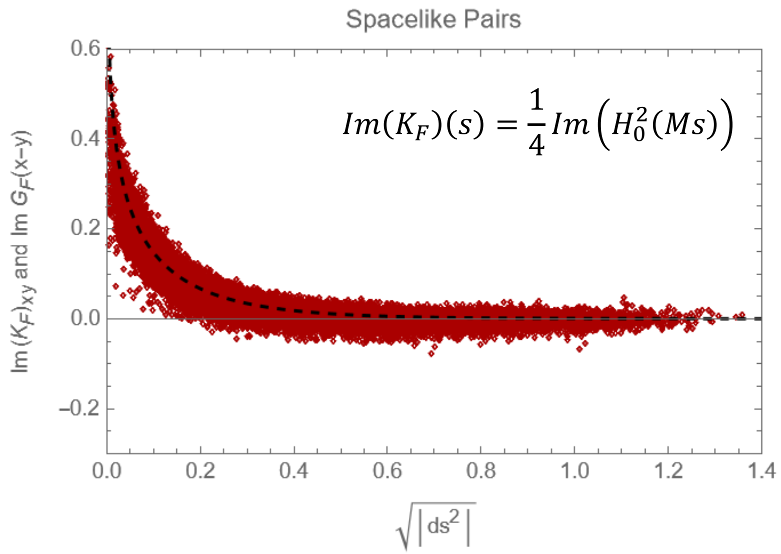
$$\boxed{i\Delta = Q - Q^* \rightarrow Im(Q) = \frac{\Delta}{2}}$$

The Feynman Propagator in CST (cont.)

$$\lim_{\epsilon\to 0}\frac{1}{(2\pi)^2}\int_{\mathbb{R}^2}\frac{1}{p_0^2-p_1^2-M^2+i\epsilon}e^{-i(p_0x_0-p_1x_1)}d^2p=\frac{1}{4}H_0^2\Big(M\underbrace{\sqrt{x_0^2-x_1^2}}_{s}\Big)$$

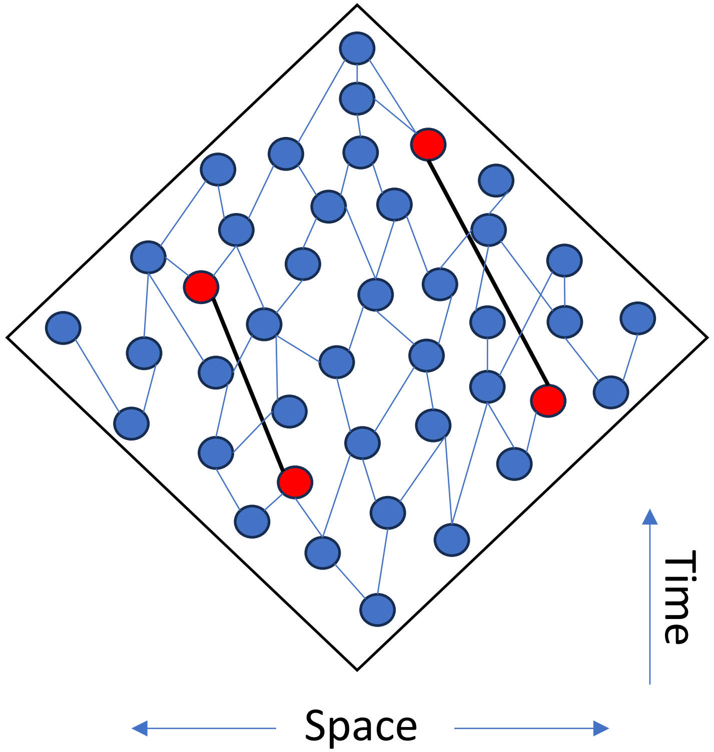






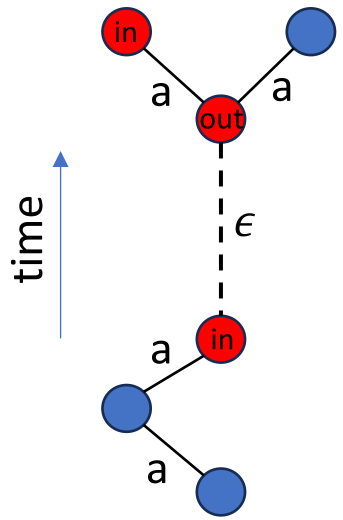
Spacetime Defects

- Several proposed quantum gravity theories postulate a discrete spacetime connected by links to form a graph
- Emergence of a manifold from a graph will generically have nonlocal defects which need not respect macroscopic locality
- A particle which encounters a nonlocal defect will experience a spacetime translation
- The defect density introduces a larger length scale than the discreteness scale

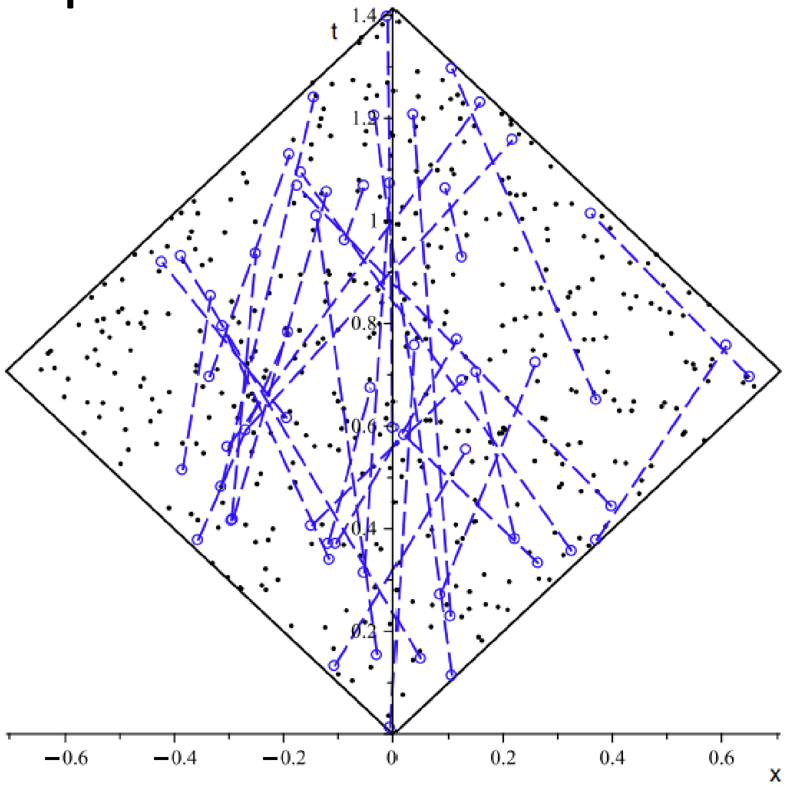


Model A

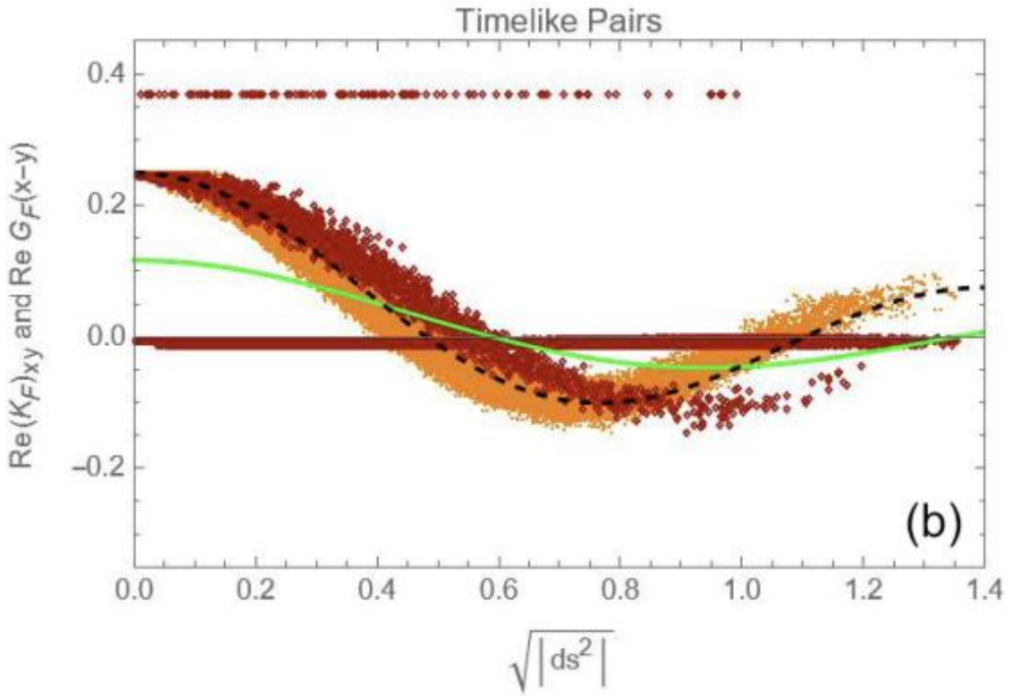
- Unit volume causal diamond in 1+1 Minkowski space
- 400 spacetime points and 200 paired defects
- In-defects and out-defects are timelike paired
- We chose $\epsilon = 0.75$
- $M = 5$

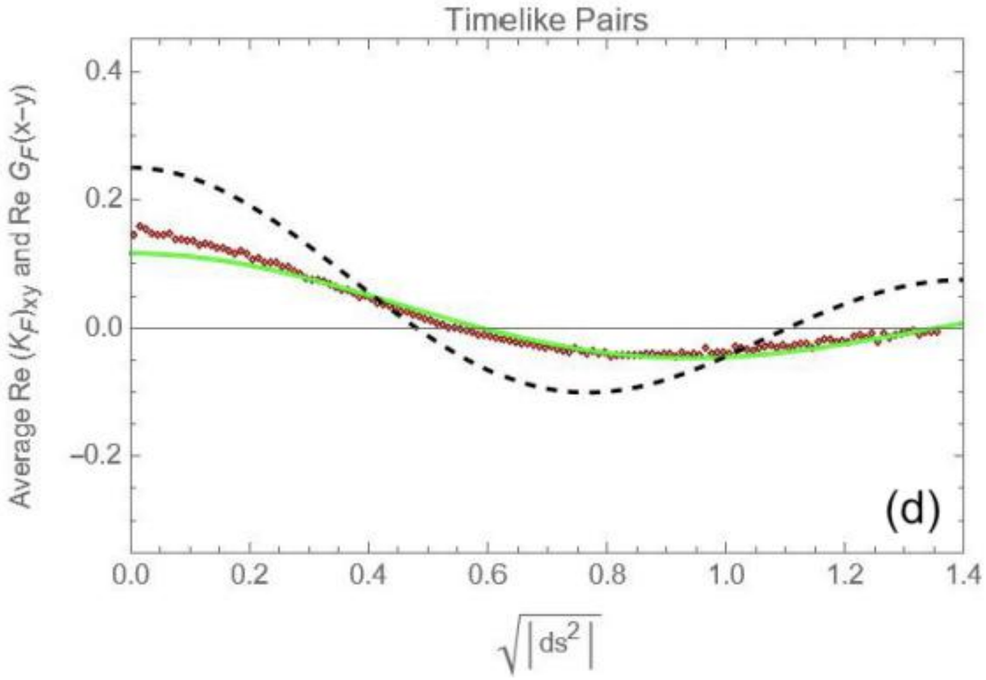


$$C = \begin{array}{c} \begin{array}{ccc}\text{Spacetime point} & \text{Out-defect} & \text{In-defect}\end{array} \\ \left(\begin{array}{c|c|c} c_{11} & 0 & c_{13} \\ \hline c_{21} & 0 & c_{23} \\ \hline 0 & \epsilon/a \cdot \mathbb{1} & 0 \end{array}\right) \end{array} \begin{array}{l} \text{Spacetime point} \\ \text{Out-defect} \\ \text{In-defect} \end{array}$$



Real Part of Feynman Propagator (A)

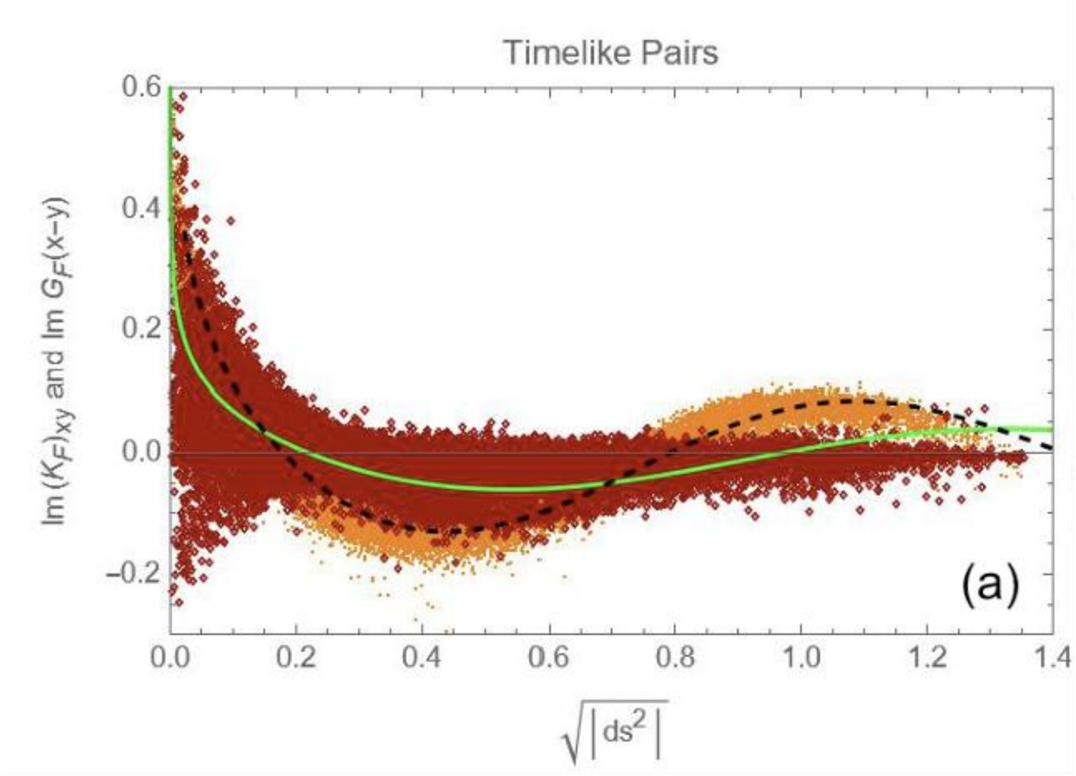


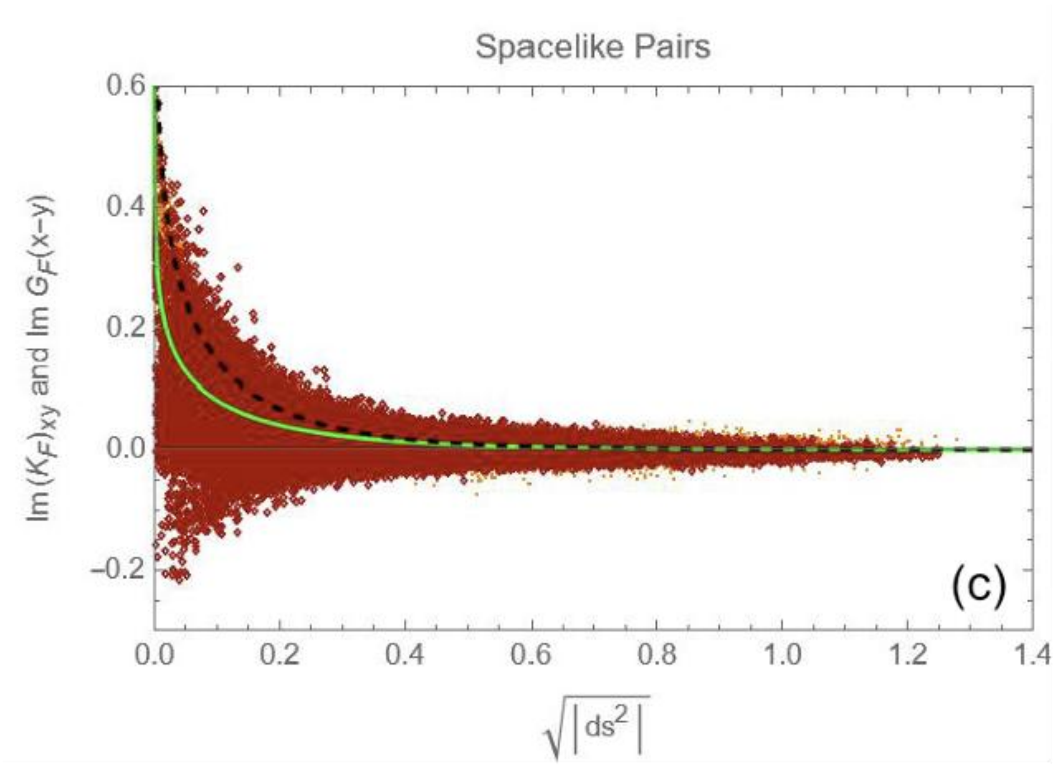


$$F(s) = \frac{1}{4} H_0^2(Ms) \rightarrow F_r(s) = \frac{Z}{4} H_0^2(M_r s)$$

$$(M, Z) = (5\,, 1) \rightarrow (M_r, Z) = (4.08\,, 0.47)$$

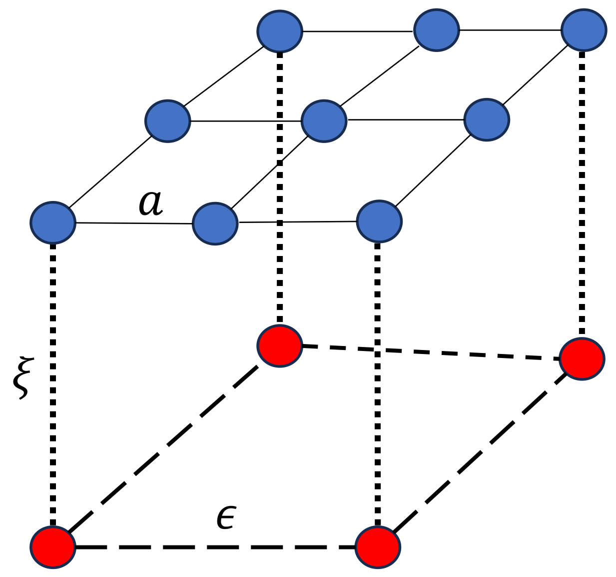
Imaginary Part of Feynman Propagator (A)





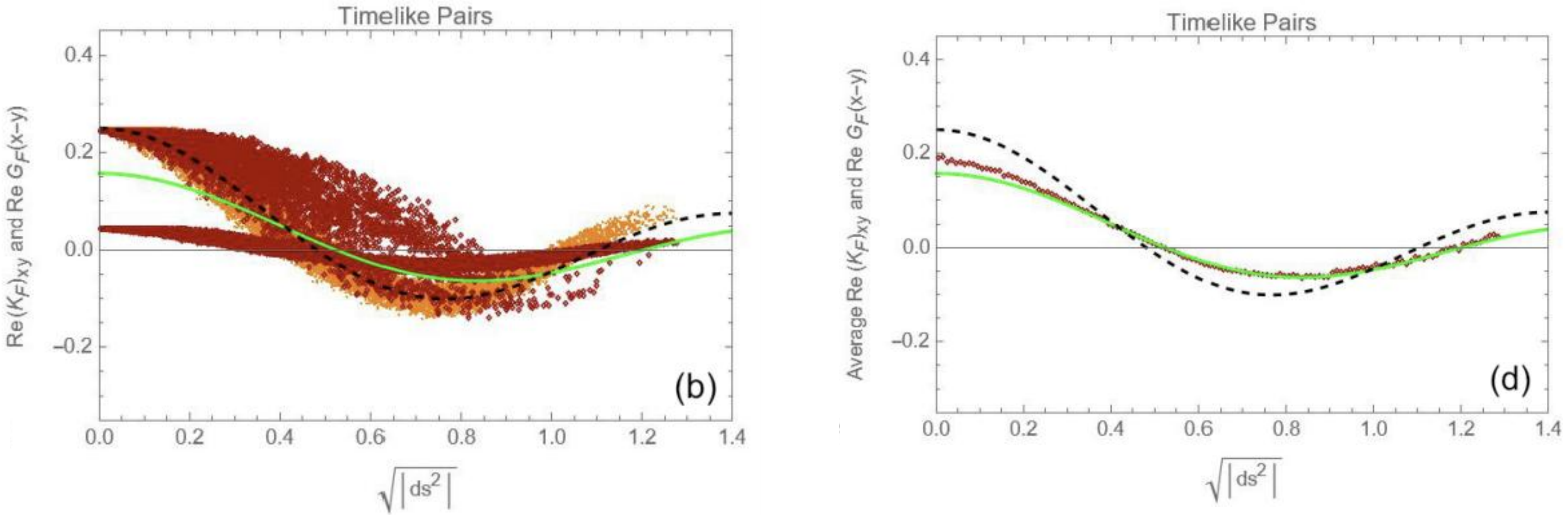
Model D

- Consider two weakly coupled lattices
- 400 spacetime points and 100 defects are Poisson distributed in a unit volume causal diamond
- Set $\epsilon = a = 0.5$ as the amplitude between defects
- Set $\xi = 0.1$ as the amplitude between a spacetime point and defect pair
- $M = 5$



$$
\begin{array}{cc}
 & \begin{array}{cc} \text{Spacetime point} & \text{Defect} \end{array} \\
C = & \left(\begin{array}{c|c} c_{11} & \xi/a \cdot c_{12} \\ \hline \xi/a \cdot c_{21} & \epsilon/a \cdot c_{22} \end{array}\right) \begin{array}{l} \text{Spacetime point} \\ \text{Defect} \end{array}
\end{array}
$$

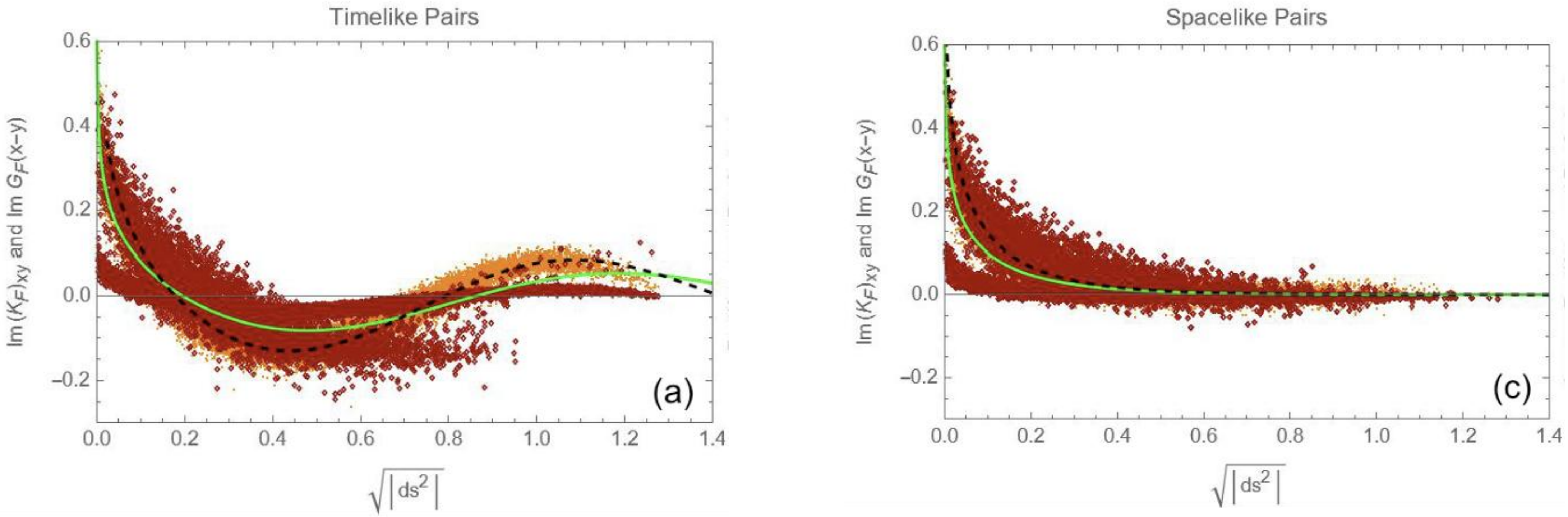
Real Part of Feynman Propagator (D)



$$F(s) = \frac{1}{4} H_0^2(Ms) \rightarrow F_r(s) = \frac{Z}{4} H_0^2(M_r s)$$

$$(M, Z) = (5\,, 1) \rightarrow (M_r, Z) = (4.59\,, 0.63)$$

Imaginary Part of Feynman Propagator (D)



Summary of Findings

- We find on average results can be described as shifts in mass and wavefunction renormalization of the continuum low-energy theory

Model	M_r/M_0	Z/Z_0	σ_M	σ_Z	R_a^2	R_b^2	R_c^2
A	0.818	0.564	0.005	0.051	0.9214	0.9474	0.8104
B	0.827	0.643	0.002	0.058	0.9487	0.9775	0.8865
C	1.000	0.999	0.004	0.086	0.9765	0.9883	0.9786
D	0.958	0.864	0.002	0.071	0.9784	0.9727	0.9348
E	0.964	0.883	0.002	0.073	0.9692	0.9728	0.9325

$$< \widetilde{K_R} > (p) = -\frac{2a}{(p_0 + i\epsilon)^2 - p_1^2 + 2ab\rho}$$

$$Re(K_F) = \frac{1}{2}\left(K_R + K_R^T\right)$$

- M_0 and Z_0 are mass and wavefunction renormalization fit parameters in the defect free limit
- Consider the average amplitude of an edge in the causal set, $\bar{a}$. Without defects, $\bar{a} = \frac{1}{2}$. If defects are present, this need no longer be true
- If $\bar{a}$ increases, so does Z and M_r. If $\bar{a}$ decreases, so does Z and M_r

Further Conclusions

- In principle, a particle collider could see the different branches of the Feynman propagator with sufficient resolution
- Our analysis may provide qualitative insight into how defects affect the Feynman propagator in 3+1 dimensional spacetime, though numerical simulations are difficult in this case

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Variation of Parameters

Model	Parameter	M_r/M_0	Z/Z_0	σ_M	σ_Z
A	$\epsilon = 0.95$	0.818	0.562	0.005	0.051
A	$\epsilon = 0.55$	0.818	0.565	0.005	0.051
B	$\epsilon = 1.2$	0.829	0.641	0.002	0.058
B	$\epsilon = 0.8$	0.827	0.644	0.002	0.058
C	$\epsilon = 0.95$	1.000	0.998	0.004	0.086
C	$\epsilon = 0.55$	1.000	1.000	0.004	0.086
D	$\xi = 0.3$	0.976	0.934	0.002	0.077
D	$\xi = 0.002$	0.954	0.794	0.002	0.066
E	$\kappa = 0.3$	0.982	0.943	0.002	0.078
E	$\kappa = 0.002$	0.954	0.854	0.004	0.070