Northeastern University

Report for Experiment #1

Measurement Lab

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Abstract

The aim of this Measurement lab report is to measure lengths and masses using basic equipment such as rulers, micrometers, and scales. The data obtained will be analyzed and plotted to extract the final results, which is the density of the material the cylinders are made of (brass) and a calculated average density value of $11.81 \pm 137.84 \frac{g}{cm^3}$. The report also covers the concept of measurement error, and the rules for calculating uncertainties. Additionally, this experiment includes a second investigation where the number of background radioactive counts per minute will be measured using a geiger counter for a final calculated average background rate of 18.32 counts/60s per our 60 experimental trials. The report emphasizes on the importance of collecting multiple data points and taking averages to improve the accuracy of the measurement.

Introduction

In this first part of the investigation, the density of a metal is determined by measuring the mass and volume of four brass cylinders. The precision of the measurements is limited by the equipment used and the care taken in obtaining the data. Measurement error is also taken into account, as it is an inherent part of any measurement. There are three main types of errors in scientific measurements: instrumental errors, systematic errors, and random errors. Instrumental errors are caused by the limitations or defects of the measuring instruments or equipment (calibration errors or wear and tear on equipment). Systematic errors are caused by a consistent bias or systematic deviation in the measurement process (incorrect measurement techniques or observer bias). Random errors are caused by unpredictable and uncontrollable variations in the measurement process (environmental conditions, measurement noise, or human error).

Density (ρ) is a measure of the mass per unit volume of a substance and is commonly measured in units of grams per cubic centimeter ($\frac{g}{cm^2}$). In this experiment, a digital scale is used to measure each mass and a 100 mL graduated cylinder filled with ~ 50 mL of water is used to calculate each volume of the four cylinders. Once you have the mass and volume measurements, you can divide the mass by the volume to find the density of the substance (density (ρ) $= \frac{mass\ (m)}{volume\ (V)}$).

The experiment also seeks to uncover the calculation of uncertainties and the propagation of errors in derived quantities. The second part of the investigation in the measurement experiment involves measuring the number of background radioactive counts present in the lab per minute with a geiger counter.

Background radiation is the level of ionizing radiation present in the environment from natural sources such as cosmic rays, radioactive elements in the soil, and radon gas. The count rate is a measure of the number of ionizing particles or photons detected per unit time, typically measured in counts per second (cps), or in the case of this experiment, counts per minute (cpm). To measure background radiation count rate, a detector such as a Geiger-Mueller (GMC-320) counter is placed in the location of interest, our 311 Churchill lab room, and the device is calibrated and started in its recording of 60 trials of cpm data for our experimental analysis.

The goal of this second part of the investigation is to quantify the uncertainties associated with random errors and calculate uncertainties in the average values using calculations of standard deviation and standard error of the mean (SEM). Standard deviation is a measure of the spread of a set of data. It quantifies the amount of variation or dispersion of a set of data values. The standard deviation is calculated by taking the square root of the variance, which is the average of the squared differences of the data points from the mean by using the equation below:

$$\delta n_{RMS} = \boldsymbol{\theta} = \sqrt{\frac{(n_1 - ave.\ of\ n)^2 + (n_2 - ave.\ of\ n)^2 + \dots + (n_N - ave.\ of\ n)^2}{N}}$$

SEM is a measure of the variability of the mean of a set of sample data. It gives an idea of the precision with which the mean of the sample estimates the true mean of the population. It is calculated as the standard deviation of the sample divided by the square root of the sample size. The SEM is often used to construct a confidence interval around the sample mean, which can be used to estimate the range of values that the true population mean is likely to fall within which was calculated with the equation below:

$$\delta(average)n = \frac{\theta}{\sqrt{N}}$$

Investigation 1

The first part of this investigation required the utilization of four brass cylinders of differing sizes and lengths, a standard ruler to measure the length and diameter of the cylinders, a digital scale to measure their masses, and a 100 mL graduated cylinder to calculate the volumes of the four cylinders.

In the first step of investigation 1, my lab partner and I used the digital scale to determine the masses (m) of each cylinder and their respective errors (Table 1.1). The scale's error was given on the physical scale as 0.10 g. This value was used to complete the error of each of their masses in the table below. In the next step, we used the standard ruler to measure the lengths (L) and diameters (D) of the four cylinders and recorded them in the table (Table 1.1). And in order to calculate the error of these two properties, we were instructed to find the smallest value on the ruler (0.10 cm) and divide it by 2 ($\frac{0.1}{2} = 0.05\ cm$) to find an error of 0.05 cm. These two measured values were then used to compute the volumes (V) using equation V = $\frac{\pi D^2 L}{4}$ and the densities (ρ) using equation ρ = $\frac{m}{V}$ of the four cylinders and their corresponding relative errors to be reported in the table found below:

Table 1.1 Measurements and Calculations for Investigation 1.

Cylinder	#1	#2	#3	#4
m (g)	3.70	6.10	14.00	21.70
δm (g)	0.10	0.10	0.10	0.10
$\frac{\delta m}{m}$	2.70	1.60	0.71	0.46
L (cm)	4.40	3.10	3.90	6.40
δL (cm)	0.05	0.05	0.05	0.05
$\frac{\delta L}{L}$	1.10	1.60	1.28	0.78
D (cm)	0.30	0.45	0.65	0.60
δD (cm)	0.05	0.05	0.05	0.05
$\frac{\delta D}{D}$	16.67	11.11	7.69	8.33

V (cm^3)	0.31	0.49	1.29	1.81
δV (cm^3)	10.34	10.92	19.91	30.33
$\frac{\delta V}{V}$	33.36	22.28	15.43	16.76
ρ($\frac{g}{cm^3}$)	11.94	12.45	10.85	11.99
δρ($\frac{g}{cm^3}$)	398.26	277.46	167.46	200.92
$\frac{\delta\rho}{\rho}$	33.35	22.29	15.43	16.76

Next, my partner and I propagated the error of the volume calculations from the previous step by inputting each of the respective volume, density, and length into the following equation found in Appendix A [3]:

$$\frac{\delta V}{V} = \sqrt{\left(2\frac{\delta D}{D}\right)^2 + \left(\frac{\delta L}{L}\right)^2}$$

In order to verify this calculation of the error, the 100 mL graduated cylinder was used as an additional method of determining one of the brass cylinder's volumes. My partner and I used our cylinder #4 for this piece of the experiment and filled the graduated cylinder with ~ 50.0 mL of water. After dropping our cylinder #4 in the ~ 50.0 mL of water, we observed the height level of the water rise to ~ 57.5 mL and determined the value of the fourth brass cylinder to be ~ 7.5 mL in this instance. It is important to note that the error of this measurement was given on our graduated cylinder with a value of 1 mL. For the purpose of this experiment, mL and cm^3 will be used interchangeably and the volume of cylinder 4 can be recorded as 7.5 cm^3, which in comparison with our previously calculated volume of 1.81 cm^3, it can be assumed that this significant difference in calculation can be attributed to random error in my partner and my volume calculations or observation of the graduated cylinder measurements and better attention to detail should be paid in these instances in future experiments. The water immersion method is the more precise method due to the value of error being a less significant factor in the measurement (1.00 < 30.33).

Next, we used formula $\rho = \frac{m}{V}$ to calculate the densities of the four cylinders, recorded these values in Table 1.1, and propagated the errors of these calculations using formula:

$$\frac{\delta\rho}{\rho} = \sqrt{\left(\frac{\delta m}{m}\right)^2 + \left(\frac{\delta V}{V}\right)^2}$$

The four cylinders are all comprised of brass, and therefore, should all have the same density of

~ 8.470 $\frac{g}{cm^3}$ [1]. This is not the density result my partner and I experimentally calculated and this fact can be attributed, again, to the random error in our measurements. However, we were able to determine the average density of our four cylinder ρ values using the equation:

$$\text{Average } \rho = \frac{\rho_1 + \rho_2 + \rho_3 + \rho_4}{4}$$

After the average ρ was determined, 11.81 $\frac{g}{cm^3}$, the error of this value was propagated using the formula:

$$\delta\rho = \frac{\sqrt{(\delta\rho_1)^2 + (\delta\rho_2)^2 + (\delta\rho_3)^2 + (\delta\rho_4)^2}}{4}$$

for a value of 137.84 $\frac{g}{cm^3}$.

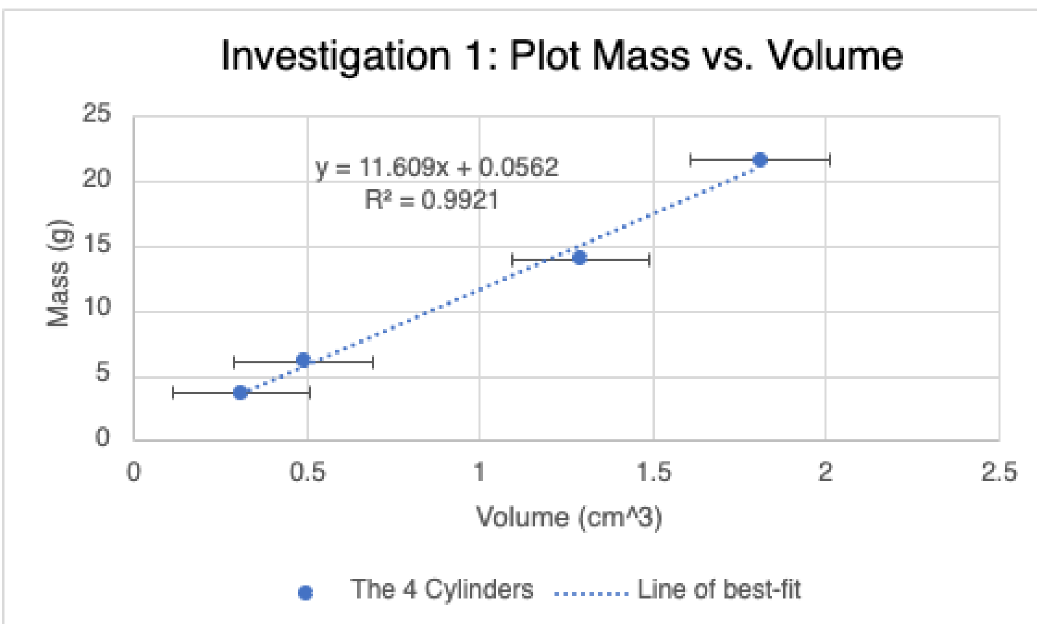

Fig. 1.2: Plot of mass vs. volume for the four brass cylinders in Investigation 1 and including the calculated line of best fit and error bars as given by Microsoft Excel.

Figure 1.2 illustrates the strong positive correlation between cylinder mass and volume in investigation 1. And because slope is calculated by $\frac{\Delta y}{\Delta x}$and ρ = $\frac{m}{V}$, the slope of the above graph is representative of the density of the four brass cylinders. However, using the IPL straight line fit calculator [4], we computed the trendline y = (11.23 ± 185.56) x + (0.34 ± 131.20), and because the slope of the trendline is equal to the average density of the four cylinders in this instance, the density = (11.23 ±

185.56). And in comparison to our previously calculated density value of 11.81 ± 137.84 $\frac{g}{cm^3}$, it is assumed that the average density determination via the provided formula and not the line of best fit is a more precise method of measurement considering the smaller margin of error (137.84 < 185.56).

Investigation 2

In the second part of this investigation into experimental measurement and error, my lab partner and I utilized the Geiger-Mueller (GMC-320) counter and GQ GMCounter PRO software to record 60 trials of background radiation count rate in cpm every 60s found below (Table 1.2). We then calculated the average count value of the 60 trials by adding up each of the 60 values and dividing them by the total number of trials, 60 (Av. = $\frac{x_1 + x_2 + x_3 + ... + x_{60}}{60}$) for a result of ~ 18.32.

Table 1.2 Measurements of counts per 60s for Investigation 2.			
Original Data		**Sorted Data (Ascending Count Value Order)**	
Trial	**Count/60s**	**Trial**	**Count/60s**
1	14	38	10
2	15	59	10
3	19	8	12
4	25	46	13
5	15	52	13
6	18	56	13
7	22	1	14
8	12	23	14
9	17	32	14
10	16	49	14
11	19	2	15
12	17	5	15
13	22	53	15
14	22	10	16

15	19	25	16
16	27	28	16
17	21	37	16
18	19	42	16
19	18	47	16
20	19	9	17
21	17	12	17
22	20	21	17
23	14	27	17
24	23	39	17
25	16	40	17
26	22	41	17
27	17	44	17
28	16	51	17
29	25	6	18
30	22	19	18
31	22	33	18
32	14	54	18
33	18	57	18
34	19	3	19
35	25	11	19
36	20	15	19
37	16	18	19
38	10	20	19
39	17	34	19
40	17	45	19
41	17	22	20

42	16	36	20
43	20	43	20
44	17	17	21
45	19	7	22
46	13	13	22
47	16	14	22
48	24	26	22
49	14	30	22
50	22	31	22
51	17	50	22
52	13	24	23
53	15	55	23
54	18	60	23
55	23	48	24
56	13	4	25
57	18	29	25
58	25	35	25
59	10	58	25
60	23	16	27

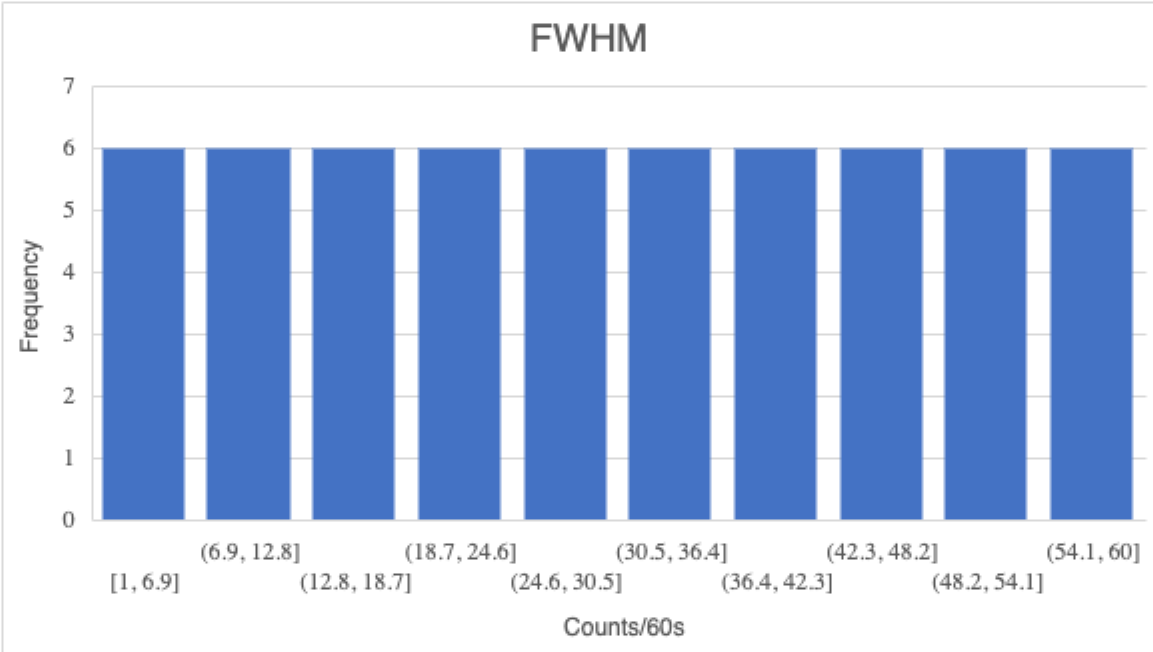

Fig 1.3: A histogram Full Width at Half Maximum (FWHM) displaying our data regarding the number of background cpm over an hour separated into 10 equal bars as given by Microsoft Excel.

We sorted the original count/60s data values into an ascending order from least to greatest in order to organize our geiger results into a histogram and calculated our bin size of 1.7 seconds given the advised division from the lab manual [3] of the data into 10 bars and using the equation:

$$\text{bin size} = \frac{(max - min)}{\#\ of\ bars}$$

We use the creation of our histogram to calculate the uncertainty (δn) of 7.23 counts using the formula below:

$$\delta n = \frac{W}{2\sqrt{2\ ln\ 2}} \approx \frac{W}{2.3548}$$

We then compared our results with our lab partner neighbors and agreed that our histogram accurately depicts the data in our table; however, the shape of it is odd and unusual. We confirmed that we accurately inputted the 60 data measurements collected from the Geiger-Mueller (GMC-320) counter and GQ GMCounter PRO software, but it is , and potentially due to random error, the histogram generated 10 equally distributed bars to represent the 6 groupings of count data. Nonetheless, our neighboring group agreed with my lab partner and I in our process of calculating the uncertainty (δn) and got the same value of 7.23 counts.

Next, we calculated the standard deviation root-mean-square deviation (δn_{RMS}), another way to measure the spread of the data, was calculated to be 3.8729 counts by using the following equation:

$$\delta n_{RMS} = \boldsymbol{\theta} = \sqrt{\frac{(n_1 - ave.\, of\, n)^2 + (n_2 - ave.\, of\, n)^2 + \dots + (n_N - ave.\, of\, n)^2}{N}}$$

Thus, the δn_{RMS} spread determined is larger than the FWHM determined. And the SEM was then calculated to be 0.4995 with the equation below:

$$\delta(average)n = \frac{\boldsymbol{\theta}}{\sqrt{N}}$$

Conclusion

This measurement experiment involved the execution of two investigations, both of which provide a recipe for determining how errors in measurement propagate to determine resulting errors in derived quantities. The purpose of investigation 1 of this experiment is to measure lengths and masses using basic equipment, and use the data obtained to analyze, graph, and determine the density of the cylinders' material.The experiment also focuses on understanding the concept of measurement error and uncertainty. Four brass cylinders mass and length were measured and used to later calculate their volume and density, all with the error of measurement at the forefront of the analysis.

One method of density determination, using the IPL straight line fit calculator [4] where average density was represented by the slope of the graphed line (Figure 1.2), resulted in a value of 11.23 ± 185.56 $\frac{g}{cm^3}$. However, the previously concluded more precise method of calculating average density output lead to a value of 11.81 ± 137.84 $\frac{g}{cm^3}$, which due to the random error in my partner and my measurements and calculations, is ~ 3.34 $\frac{g}{cm^3}$ off of the known density of brass ($8.47 \frac{g}{cm^3}$).

The report emphasizes the importance of repeating measurements and taking averages to improve the accuracy of the data obtained. However, derived quantities, such as the volume of the cylinder, had errors that were propagated from the measurements of diameter and length. Various equations were given (i.e the volume error propagation formula: $\frac{\delta V}{V} = \sqrt{(2\frac{\delta D}{D})^2 + (\frac{\delta L}{L})^2}$) and utilized to account for this propagation in order to maximize the precision of our experiment trials and all values were recorded in Table 1.1; however, the accuracy of our information could have been improved upon by executing multiple trials of our measurements for average values that would minimize the random error in our original trial effort.

Whereas, in investigation 2 of this measurement experiment, the number of background radioactive counts present in the lab per minute will be measured using a Geiger-Mueller (GMC-320) counter and GQ GMCounter PRO software. The counter and software were set up and administered on the desktop computers provided in our lab room and an hour of cpm data was collected (Table 1.2). Through this process of data collection, it's important to note that the detector was calibrated before use and measurements were collected over a period of an hour, long enough to account for variations in the count rate.

The data collected was recorded carefully, but the most important step was the data analysis, where calculations were made to extract the final results. The cpm data was organized in ascending order and organized into a histogram (Figure 1.3) to better visualize our collected results. In our trials, it became clear to us that the most common type of errors we encountered were random errors, which were quantified using the standard deviation root mean and the standard error in the mean value of the data. Meaning a lower standard deviation is optimal and correlates with minimal numerical outliers and a more normalized spread of the data. Our calculated value of standard deviation root-mean-square deviation, 3.87 counts, and SEM, 0.50, demonstrate how our data is varied approximately 4 number values away from the average count/60s value collected.

The goal of this experiment was to quantify the uncertainties associated with these errors, and provide an opportunity to practice calculating uncertainties when random errors are present. But it could be argued that in both investigations, the precision of measurements was limited by the equipment used and the care taken in obtaining the data, and all measurements had an inherent uncertainty of measurement error. Overall, investigation 1 and 2 of this measurement experiment could benefit from increasing the amount of trials to diversify and expand upon the sample size and inhibit the executionary error.

Questions

1. **If you had forgotten to zero-out (tare) the scale before weighing the cylinders in Investigation 1, how would it have affected your data? What type of error would this have introduced into your calculations?**

   If the scale had not been tared before measuring the cylinders in Investigation 1, it would have affected the data by introducing systematic error into the measurements of mass. This type of error would have resulted in all of the measured masses being off by the same amount, which would have affected the calculated density and introduced uncertainty into the results. This error would have been caused by the scale not being properly calibrated and would have been a consistent bias, affecting the accuracy of the measurement.

2. **A cylinder of the same material as the one you used in your experiment has a mass of 250 g and a diameter of 10 cm. What is its length?**

   m = 250 g, d = 2r = 10 cm, therefore, r = 5 cm, $\rho = 8.470 \frac{g}{cm^3}$

   $V = \pi(\frac{D}{2})^2 L$, m = (ρ)(V)

   $250\ g = (8.470 \frac{g}{cm^3})\ (\pi\ (\frac{10cm}{2})^2 L)$

   $L = \frac{250\ g}{(8.470 \frac{g}{cm^3})\pi(25\ cm)^2}$ = **1.18 cm**

3. **A sphere of the same material as the one you used in your experiment has a radius r = 10 cm. What is its mass? (Hint: $V_{sphere} = \frac{4}{3}\pi r^3$)**

V = $\frac{4}{3}\pi(10cm)^3$ = 4,188.79cm^3; m = (8.470 $\frac{g}{cm^3}$)(4,188.79cm^3) = 35,289.4 g = **35.29 kg**

4. **Suppose you receive a traffic ticket for speeding and want to contest it in court. Come up with two arguments, one using systematic error and the other using random error, that you could use to challenge the speed given by either your speedometer or the radar gun.**

   In terms of systematic error, it could be argued that the speedometer in your car was calibrated incorrectly, leading to a consistently higher reading than your actual speed. This is an example of a systematic error, as it would affect all measurements taken with the speedometer in the same way. In terms of random error, it could be argued that the radar gun used by the officer was not properly maintained, leading to inaccurate readings. This is an example of random error, as it would affect measurements taken with the radar gun in a random and unpredictable way.

5. **If the data from two Geiger counters are combined, how will the standard deviation of the new data set compare to that of each of the individual Geiger counters?**

   If the data from two Geiger counters are combined, the standard deviation of the new data set will likely be smaller than that of each of the individual Geiger counters. The standard deviation of the combined data set can be calculated and then used to calculate the standard error in the mean value. This is because taking the average of multiple measurements reduces the overall measurement error by decreasing the random scattering of the individual measurements and decreases the overall spread of the data, resulting in a smaller standard deviation. However, it is important to note that this would depend on the specific measurements and data collected. It is an experimental best practice to calculate the uncertainties associated with the measurements and the derived quantities to determine the magnitude of the errors.

Honors Question

3. In honors question 2, how many additional measurements will you need to take to decrease the error in the mean by half? Given = 20 trials.

Error = $\frac{1}{\sqrt{N}}$

Error = $\frac{1}{\sqrt{20}}$, $(Error)^2 = (\frac{1}{\sqrt{20}})^2 = \frac{1}{(20)(4)} = \frac{1}{N}$, N = 80

There would need to be an additional 80 measurements to decrease the error in the mean by half.

Acknowledgments

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