Northeastern University

Report for Experiment #5

Uniform Circular Motion

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Abstract

For this experiment, we conducted a circular motion analysis by spinning a bob with varying masses attached to it and utilized a Sargent-Welch centripetal force apparatus to determine the centripetal acceleration of the objects. The centripetal forces of the six apparatus revolution trials of differing tensions led to centripetal force calculations of 0.839 ± 0.326 N, 1.650 ± 0.422 N, 2.606 ± 0.351 N, 15.360 ± 1.210 N, 3.810 ± 0.357 N, and 1.400 ± 0.229 N respectively. Our main objective was to comprehend the effects of acceleration, velocity, and period on centripetal force, while taking into account the possibility of error propagation. We applied centripetal force to the bob using rubber bands of varying tensions to explore their impact on centripetal acceleration and force as the bob circled the apparatus. We presented our findings through two graphs that depicted the relationships between force versus velocity and force versus velocity squared, uncovering positive slope relationships in both plots.

Introduction

This experiment aims to investigate rotational motion, which involves an object moving in a circle that changes its direction continually, resulting in a change of the velocity vector direction, with the magnitude staying constant. As a result of the continuous change in direction, the object experiences centripetal acceleration. This experiment explores the relationship between the acceleration, velocity, and force on an object in uniform circular motion. In order to calculate an object's centripetal force, we applied Newton's Second Law of Motion to uniform circular motion. We determined the values of force (F), velocity (v), and radius (r) and analyzed the data numerically and graphically.

The Sargent-Welch centripetal force apparatus (Figure 1.1), rubber bands, timer, string, paper clips, a bucket, washers, a ruler, and a scale are all utilized in a collection of six trials to gather data. The mass of the bob attached to the apparatus is measured, and by varying the rubber band strength, we recorded the time for two rotations while ensuring the bob is directly over the pointer as the instance of each time (seconds) collection.

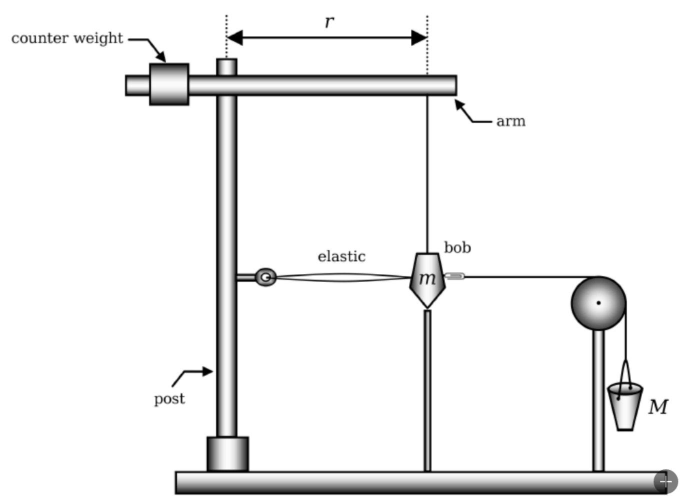


Figure 5.4 Sargent-Welch centripetal force apparatus arranged to measure the force exerted by the elastic. At this time, the bob is not rotating. The rubber band (or spring) has been stretched by adding washers to the bucket, increasing its mass *M* until the bob is aligned with the pointer. Before rotating the arm, the mass *M* and string are removed. The speed of rotation is increased until the bob is again aligned with the pointer. The elastic, stretched to the same length as before, now provides the centripetal force maintaining the circular motion of the bob. The magnitude of this centripetal force is $F = Mg$.

Fig. 1.1: The components and function of the Sargent-Welch centripetal force apparatus [2].

The uncertainties, deviation, and average values of the recorded data are also calculated and the results of the six trials are analyzed and displayed graphically to draw conclusions on the velocity and acceleration of an object in uniform circular motion and the relation and importance of angular velocity to linear velocity. Overall, this experiment will provide valuable insights into rotational motion and its underlying principles.

Investigation 1

To conduct this uniform circular motion experiment, we utilized a Sargent-Welch centripetal force apparatus (Figure 1.1), rubber bands, a timer, assorted springs (four of differing tensions), string, paper clips, a bucket, washers, a ruler, and a scale. Firstly, the mass of the bob attached to the apparatus was weighed and recorded (Table 1.1.1 and Table 1.1.2), the error of this measurement was provided by the scale to be 0.10g or 0.0001kg. Next, the table was leveled and adjusted to the apparatus so that the pointer of the apparatus was resting a few millimeters below the bob and directly below the middle of the motionless bob for each of the spring trials. The radius, measurement specifics detailed in Figure 1.1, of our centripetal force apparatus arm was measured and recorded in the tables provided below. The error of this measurement was provided by our lab ruler as half .1 cm or 0.0005m.

Then, a rubber band with two paper clips attached to it was secured to the post and the bob in such a manner that when the apparatus lay stationary, the bob was pulled towards the vertical post by the rubber band. Next, we calculated the force necessary to bring the bob back over center the pointer using a string tied to the bucket attached to the bob-counter structure and then adding in washers into the bucket until the bob was re-aligned with the pointer (Figure 2.1). Then, the washer-filled bucket and string were removed and the mass M were recorded. The washer-bucket-string combination was re-attached as before to find the error of this M measurement by adding additional washers until the bob is pulled past center over the pointer. This new, heavier, M is put on the scale and recorded. Next, the bucket and string are removed and a timer is set up. My lab partner twists the post to begin the rotation of the Sargent-Welch apparatus with one rubber band attached and the time it takes to complete two rotations over center is recorded. This concludes the experimental testing processes of Trial 1. These recorded values are used to calculate the other values in the Circular Motion Data Tables.

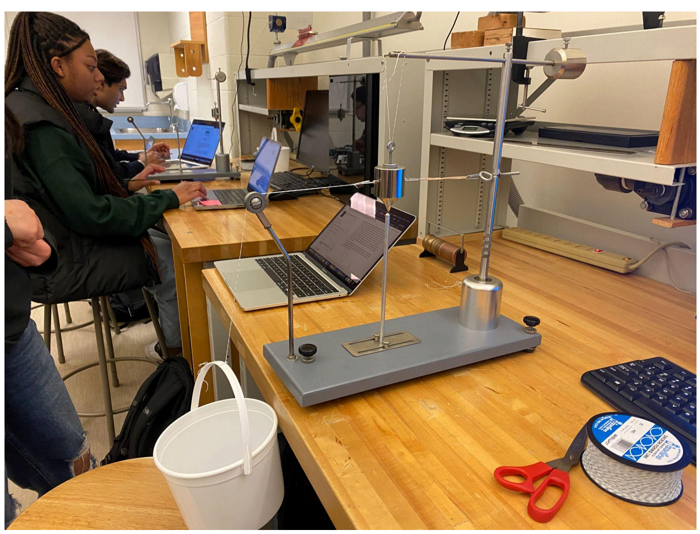


Fig. 2.1: My lab partner and my string-bucket apparatus.

Then, this same Trial 1 process is repeated for Trials 2, 3, 4, 5, and 6 and the results are recorded. Trial 1, as aforementioned, was calculated with the tension of one rubber band attached to the bob and post, Trial 2 was performed with two rubber bands attached, Trial 3 with the spring requiring mass of approximately 300g attached to the bob and post, Trial 4 with the spring requiring mass of approximately 600g, Trial 5 with the spring requiring mass of approximately 400g, and Trial 6 was performed with the spring requiring mass of approximately 200g attached to the bob and apparatus.

Table 1.1.1: Circular Motion Data Table

Mass of the bob = 0.454 (kg), δm (kg) = 0.0001 **Radius of the apparatus, r = 0.185 (m), δr (m) = 0.0005**

Trial #	M (kg)	δM (kg)	Centripetal force, F = Mg (N)	δF (N)	# of revolutions	t_1 (s)	t_2 (s)	t_3(s)	Average time, $\bar{t}$ (s)	$\delta\bar{t}$ (s)
1	0.086	0.033	0.839	0.326	2	5.18	6.09	3.89	5.053	0.782
2	0.169	0.043	1.650	0.422	2	3.18	2.82	3.01	3.003	0.127
3	0.266	0.036	2.606	0.351	2	3.5	3.73	4.08	3.770	0.207
4	1.568	0.124	15.360	1.210	2	3.23	3.5	4.07	3.600	0.303
5	0.389	0.036	3.810	0.357	2	2.71	3.07	3.31	3.030	0.214
6	0.142	0.023	1.400	0.229	2	3.34	3.84	3.87	3.683	0.211

Table 1.1.2: Circular Motion Data Table

Mass of the bob = 0.454 (kg), δm (kg) = 0.0001 **Radius of the apparatus, r = 0.185 (m), δr (m) = 0.0005**

Trial #	Period T (s)	δT (s)	v $(\frac{m}{s})$	δv $(\frac{m}{s})$	v^2 $(\frac{m}{s})^2$	δv^2= 2vδv $(\frac{m}{s})^2$
1	2.520	1.563	0.0012	0.1547	0.00000144	3.71 x 10^-4
2	1.500	0.255	0.002	0.0424727	0.00000400	1.7 x 10^-4
3	1.890	0.413	0.0017	0.05484	0.00000289	1.86 x 10^-4
4	1.800	0.606	0.0017	0.0842655	0.00000289	2.87 x 10^-4
5	1.520	0.427	0.0021	0.070513	0.00000441	2.96 x 10^-4
6	1.840	0.210	0.0017	0.05721	0.00000289	1.94 x 10^-4

The values collected from our six differing tension scenario trials were used to calculate the centripetal force, F, using F=Mg with g = 9.81. For example in Trial 1, F= (.0856)(9.81)= 0.839 N. The error of this measurement was calculated with equation δF= |g|(δM) by multiplying the error of the M measurement by g, δF= (δM)(9.81) in Newtons (N). The F of the six trials were calculated to be 0.839 $\pm$ 0.326 N, 1.650 $\pm$ 0.422 N, 2.606 $\pm$ 0.351 N, 15.360 $\pm$ 1.210 N, 3.810 $\pm$ 0.357 N, and 1.400 $\pm$ 0.229 N respectively. Three time trials of the two revolutions for each were recorded and the average of each of the revolution times were averaged, $\bar{t}\,(s) = \frac{t_1 + t_2 + t_3}{3}$, for the six trials respectively. The error of the average time was determined using equation:

$$\delta\bar{t} = \frac{\sigma}{\sqrt{N-1}}, \text{ where } \sigma = \sqrt{\frac{(t_1-\bar{t})^2 + (t_2-\bar{t})^2 + (t_3-\bar{t})^2}{N-1}}.$$

The period, T, was calculated with the equation, $T = \frac{\bar{t}}{\#\ of\ revolutions}$, where the denominator, in this instance, was 2 each trial. The error for this calculation was also calculated and inputted in the Circular Motion Data Table using equation:

$$\delta T = \frac{\delta\bar{t}}{\#\ of\ revolutions}.$$

Next, the velocity, v, was computed from the values located in the table with equation $v = \frac{2\pi r}{T}$. For example, using Trial 2's data, $v = \frac{(2)(\pi)(0.185)}{1.500} = 0.002\frac{m}{s}$. The error of the velocity was then calculated using the following equation:

$$\frac{\delta v}{v} = \sqrt{\left(\frac{\delta r}{r}\right)^2 + \left(\frac{\delta T}{T}\right)^2}.$$

And lastly, the $velocity^2$ was determined mathematically by squaring the previously determined velocity values and their respectives errors were calculated using equation:

$$\delta v^2 = (2)(v)(\delta v).$$

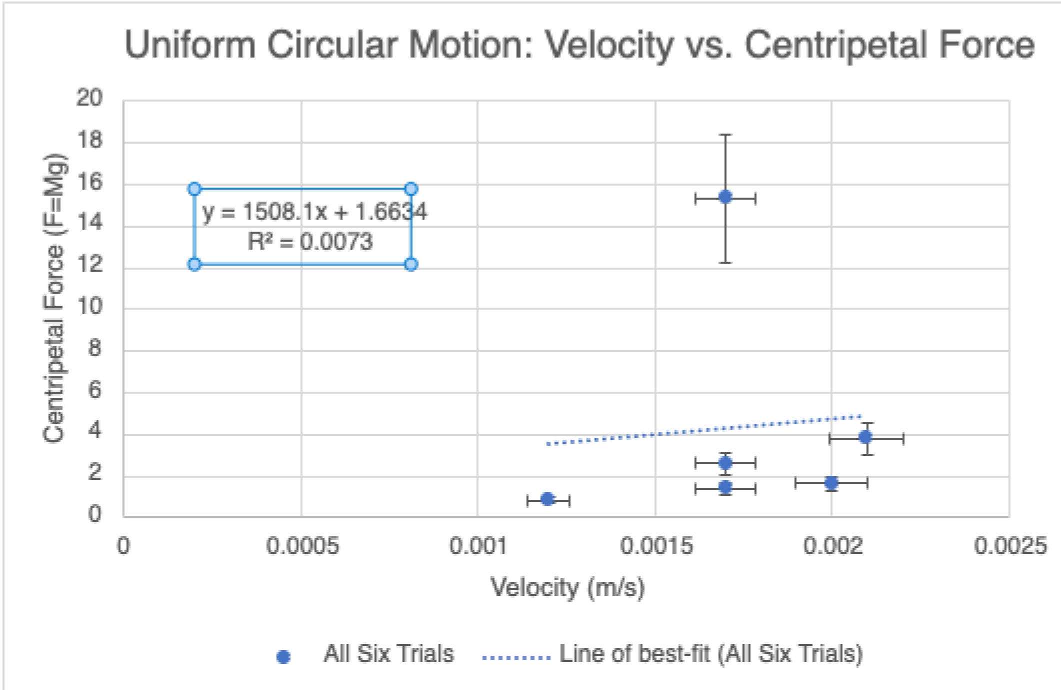


Fig 2.2: Plot of Centripetal Force vs. Velocity for the six trials.

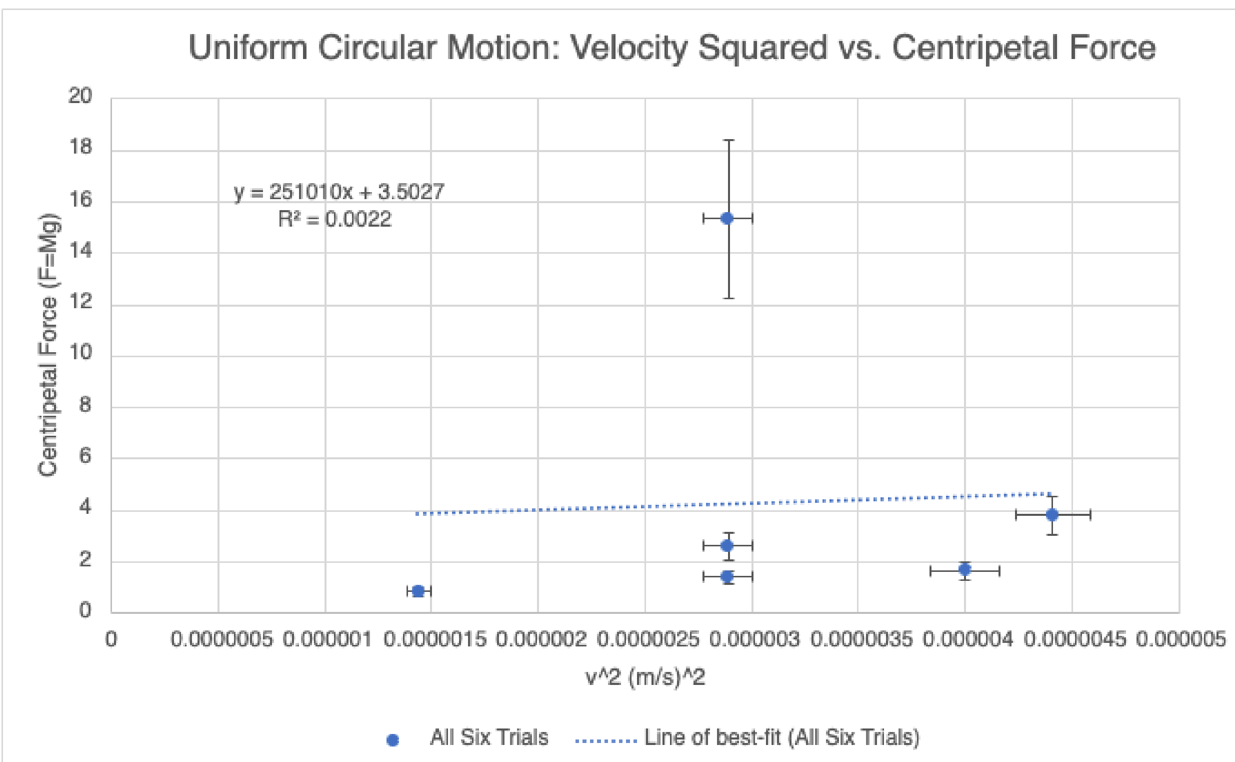


Fig 2.3: Plot of Centripetal Force vs. $Velocity^2$ for the six trials.

The centripetal force, F, and velocity, v, values for the six trials were then displayed graphically (Figure 2.2) to analyze and learn more about their corresponding relationship, the aforementioned calculated errors, δF and δv were utilized as the displayed error bars in the above plot. The slope ($\frac{kg}{m}$) of the graph, 1508.1 $\pm$ 38.149, is positive as expected demonstrating that an increase in velocity results in an increase in centripetal force required to keep the object moving in a circle. In terms of physics, a positive slope on this graph is an indication that the centripetal force required to maintain circular motion is directly proportional to the velocity of the object.

The centripetal force, F, and $velocity^2$, v^2, values were also plotted (Figure 2.3) to display visually their relationship, the aforementioned calculated errors, δF and δv^2, were used to construct the displayed error bars on the above graph. The second plot's slope, 251010 $\pm$ 162.379, displays a positive slope relationship between F and v^2. This relationship is described by the equation $F_c = \frac{(m)(v^2)}{r}$, where F_c is the centripetal force, m is the mass of the object, v is its velocity, and r is the radius of the circular path. An experiment that produces a graph with a positive slope on the velocity-centripetal force graph would confirm this relationship between velocity and centripetal force. The two slope values were confirmed using the IPL Straight Line Fit Calculator [3].

Conclusion

In conclusion, the goal of our physics experiment was to gain a deeper understanding of the roles of acceleration, velocity, and period in centripetal force while also taking into account the concept of error and its propagation. In uniform circular motion, the velocity of an object tangent to the circle at the instantaneous position of the object changes constantly, leading to acceleration. This acceleration is directed towards the center of the circle and is referred to as centripetal acceleration. Because circular motion always involves a centripetal acceleration, Newton's Second Law requires a centripetal force. We measured the centripetal force at the beginning of each experiment using our string-bucket-washer method to find M where F = Mg, and obtained values of F of the six trials were calculated to be 0.839 $\pm$ 0.326 N, 1.650 $\pm$ 0.422 N, 2.606 $\pm$ 0.351 N, 15.360 $\pm$ 1.210 N, 3.810 $\pm$ 0.357 N, and 1.400 $\pm$ 0.229 N respectively.

During the course of our experiment, we timed the bobs for two revolutions, which introduced the possibility of human error in our measurements. This could have led to our timing for the two revolutions being greater than they should be. Additionally, the bob was supposed to be directly over the pointer, but this was difficult to ensure while the bob was in motion. In order to account for the timing error, we took three measurements for each trial and used their average. Other potential sources of error include systematic errors, such as a tilt in our apparatus or the bucket scraping against the pole, which made it challenging to determine the mass required to reset the bob to the pointer.

Furthermore in consideration of potential experimental error, it should be noted that our spring for Trial 4 required a mass over double 600g and this is to be taken into consideration when analyzing our data. It was explained by our Lab TA, Yin-Chun Hung, that the importance of distinguishing between the required mass for each of the springs was to make sure my lab partner, Kelly, and I collected data from four different tension strength springs. This was successful in our trials; however, it is important to consider how much more mass this one spring required in comparison to the other five measurements as it turns out to be somewhat of a numeric outlier in our data table and graphs. If this experiment were to be repeated by Kelly and I, we would likely opt out of including this spring in our trials due to this fact.

We acknowledge that there are potential sources of error in any experiment, and we worked to account for and minimize these uncertainties as much as possible. In order to improve the accuracy of our weight measurements, more precise or smaller weights could be used. Furthermore, updating the apparatus to ensure that it is level and the bucket does not scrape against the pole could also improve the accuracy of our measurements. Also, the calculated slopes of our two graphs, although positive as expected, were much larger than anticipated and can be attributed to a myriad of factors including random experiment executionary or calculation error by Kelly and I, the systematic error of excel in calculating these values from our inputted values and graph formatting inputs, and also, this could potentially be attributed to the aforementioned Trial 4 outlier M value.

In summary, our experiment allowed us to better understand the concept of uniform circular motion and the role of centripetal force in maintaining circular motion. When an object moves in a circle, its direction changes continually, resulting in a changing velocity vector. While the magnitude of this vector may stay constant, the continuous change in direction indicates that the object in circular motion always experiences acceleration, which is referred to as centripetal acceleration. This is the most important difference when compared to constant linear motion, which has no acceleration. The acceleration of an object moving in a circle, $a = \frac{v^2}{r}$, is always directed towards the center of the circle, and as such, there is always a centripetal force acting on the object. This force is necessary to maintain circular motion, as without it, the object would not continue to move in a circular path.

Through careful consideration of potential sources of error and their propagation, we were able to gain a clearer understanding of the physics involved in this type of motion. The objective of this experiment is to understand the velocity and acceleration of an object in uniform circular motion and the relation and importance of angular velocity to linear velocity. This knowledge could be applied in a variety of fields, including engineering, physics, and even everyday situations such as driving a car around a bend in the road.

Questions

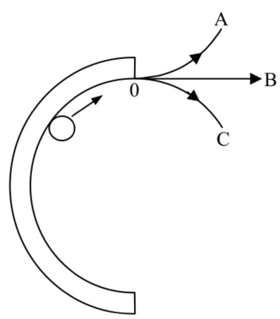
1. **What are the units of the slope of your F vs. v2 plot?**

   The units for the slope of force versus $velocity^2$ is $\frac{kg}{m}$.

2. **The lines that you draw through the data on your F vs. v2 plot should pass through the origin. Give a simple reason for this.**

   When the velocity of an object is zero, there will be no centripetal force acting on it, and the object will be at rest. This is why the line of best fit passing through the origin is significant, as it indicates that there is no centripetal force acting on the object.

3. **A marble rolls along the inside of a semicircular track made of copper tubing that is lying on a level table that is parallel to the floor (Fig. 5.6). As the marble leaves the track at point 0, does it follow path A, path B, or path C? Explain your answer in terms of the concepts developed in this experiment.**

   The marble will follow the path of B due to the centripetal force acting on it, with the normal force providing the centripetal acceleration. The centrifugal force will try to maintain the object's motion in the same direction, but once the centripetal force, in this case the track, is removed, the marble will move in a straight line. Due to gravitational force and the momentum of the marble, it will gradually fall and cannot move to paths A or C respectively, leading it to follow path B.

4. **What is the precision (relative error) of the centripetal force divided by the mass if the velocity and the radius are each determined with a precision of 1%?**

   $F = \frac{mv^2}{r}, \frac{F}{m} = \frac{v^2}{r}$

   Precision of $\frac{F}{m}$ = $2(\frac{\Delta v}{v}) + \frac{\Delta r}{r}$

   Given = $\frac{\Delta v}{v}$ = 1% or 0.01 and $\frac{\Delta r}{r}$ = 1% or 0.01

   Precision of $\frac{F}{m}$ = (2 x 0.01) + 0.01 = 0.02 + 0.01 = **0.03 or 3%**

5. **In order to keep a constant centripetal acceleration, how much should the radius change if the velocity is doubled?**

   Because $a = \frac{v^2}{r}$ and $\frac{v^2}{r} = \frac{(2v)^2}{4r} = \frac{4v^2}{4r} = \frac{v^2}{r}$, in order to keep a constant centripetal acceleration if the velocity is doubled, the radius, r, would need to be quadrupled.

Acknowledgments

I would like to thank my lab partner, Kelly Rosendo, for her help in executing this experiment during our lab class. I would also like to thank our Lab TA, Yin-Chun Hung, for his role in outlining the steps of the experimental process and answering any clarifying questions during our two investigations. Lastly, I would like to thank Northeastern University for providing me with the opportunity and resources to conduct this uniform circular motion lab and write this lab analysis.

References

[1] H. Young and R. Freedman, University Physics, p 111, Pearson Education, 14th edition.

[2] Hyde, Batishchev, and Altunkaynak, Introductory Physics Laboratory, pp 411-415, Hayden-McNeil, 2017.

[3] IPL Straight Line Fit Calculator, http://www.northeastern.edu/ipl/data-analysis/straight-line-fit/.