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Stabilization of prescribed values and periodic orbits with regular and pulse target oriented control

E. Braverman[a)] and B. Chan
Department of Mathematics and Statistics, University of Calgary, 2500 University Drive N.W., Calgary AB T2N 1N4, Canada

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Investigating a method of chaos control for one-dimensional maps, where the intervention is proportional to the difference between a fixed value and a current state, we demonstrate that stabilization is possible in one of the two following cases: (1) for small values, the map is increasing and the slope of the line connecting the points on the line with the origin is decreasing; (2) the chaotic map is locally Lipschitz. Moreover, in the latter case we prove that any point of the map can be stabilized. In addition, we study pulse stabilization when the intervention occurs each m-th step and illustrate that stabilization is possible for the first type of maps. In the context of population dynamics, we notice that control with a positive target, even if stabilization is not achieved, leads to persistent solutions and prevents extinction in models which experience the Allee effect. © *2014 AIP Publishing LLC.* [http://dx.doi.org/10.1063/1.4865231]

After it was discovered that even simple maps exhibit transition to the chaos through a series of period doubling bifurcations, some methods of control and stabilization were suggested. A two-parameter target oriented control including a linear transformation of the argument was proposed by Dattani *et al.* in 2011 and modified by Liz and Ruiz-Herrera in 2012 and applied to various models of population dynamics.

First, we extend the stabilization results of the previous papers to a more general class of maps, in particular, to non-unimodal functions that also occur in mathematical biology.

Second, we justify the possibility of stabilizing an m-periodic orbit with applying target oriented control each m-th step and illustrate the result with numerical examples one of which analyzes the change of per step control intensity as the distance $m-1$ between the steps at which the control is applied grows.

Third, we prove that for any locally Lipschitz map stabilization of any point in its range is possible with target oriented control. In fact, depending on the chosen target and control intensity, convergence to any point can be achieved, not only in the domain of the original function. A numerical example shows that theoretically found control bounds are close to the last period-halving bifurcation point after which the model is stable.

I. INTRODUCTION

A simple difference equation

$$x_{n+1} = f(x_n) \tag{1.1}$$

is known to demonstrate chaotic behaviour, see Ref. 12. Several techniques have been introduced to control and stabilize (1.1). The proportional feedback method (PF)

$$x_{n+1} = f((1-\mu)x_n) \tag{1.2}$$

consists in a reduction of the state variable, proportional to the size of this variable, where $\mu \in [0, 1)$. The assumption of the proportional reduction is aligned with the idea of constant effort harvesting, see the monograph by Clark[7] for details or pest control as in the papers by Seno[16] and Zipkin *et al.*[19] In the latter case the decrease of the average controlled population is a desired outcome which is not always the case, see the papers by Abrams,[1] Liz and Ruiz-Herrera,[11] Seno,[16] and Zipkin *et al.*,[19] and references therein. PF method is a one-parameter control method where only deductions are introduced at each step (harvesting) and the positive equilibrium of (1.2) is expected to be smaller than of (1.1). In the case when the system experiences the Allee effect first introduced by Allee,[2] see also the recent papers by Boukal and Berec[3] and Schreiber,[15] application of PF can lead to extinction. This is caused by the shift to the area of smaller population densities with lower fitness and reproduction rates due to possible difficulties in finding a mate, implement group defense, etc.

To the best of our knowledge, the system where PF was applied only at certain steps

$$x_{n+1} = \begin{cases} f(x_n), & \text{if } n \neq mk, \\ f((1-\mu)x_n), & \text{if } n = mk,\ k \in \mathbb{Z}^+, \end{cases} \tag{1.3}$$

where m is a positive integer, for the first time was proposed by Güémez and Matías.[10] In (1.3) the control is applied in the form of pulses after m iterations of f, and an m-cycle can be stabilized rather than a positive equilibrium, see Braverman and Liz[5] for global stability analysis of (1.3). However, (1.3) inherited the extinction problems of (1.2) in the presence of the Allee effect, even for large enough initial values, as was shown by Braverman.[4]

The prediction based control (PBC)

$$x_{n+1} = f(x_n) - c(f(x_n) - x_n) = cx_n + (1-c)f(x_n) \tag{1.4}$$

a)Electronic mail: maelena@ucalgary.ca

