ISSN 1855-3966 (printed edn.), ISSN 1855-3974 (electronic edn.)
ARS MATHEMATICA CONTEMPORANEA 21 (2021) #P2.03
https://doi.org/10.26493/1855-3974.2260.c0e
(Also available at http://amc-journal.eu)

A generalization of balanced tableaux and marriage problems with unique solutions

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Received 22 February 2020, accepted 24 February 2021, published online 30 September 2021

Abstract

We consider families of finite sets that we call flagged and that have been characterized by Chang as being the families of sets that admit unique solutions to Hall's marriage problem and we consider generalizations of Edelman and Greene's balanced tableaux previously investigated by Viard. In this paper, we introduce a natural generalization of Edelman and Greene's balanced tableaux that involves families of sets that satisfy Hall's marriage condition and certain words in $[m]^n$, then prove that flagged families can be characterized by a strong existence condition relating to this generalization. As a consequence of this characterization, we show that the arithmetic mean of the sizes of subclasses of such generalized tableaux is given by a generalization of the hook-length formula.

Keywords: Balanced tableaux, Hall's marriage condition, shelling.

Math. Subj. Class. (2020): 05A20, 05C70, 05E45

1 Introduction

Hall's Marriage Theorem is a combinatorial theorem proved by Hall [11] that asserts that a finite family of sets has a transversal if and only if this family satisfies the marriage condition. This theorem is known to be equivalent to at least six other theorems which include Dilworth's Theorem, Menger's Theorem, and the Max-Flow Min-Cut Theorem [20]. Hall Jr. proved [10] that Hall's Marriage Theorem also holds for arbitrary families of finite sets, where by arbitrary we mean families of finite sets that do not necessarily have a finite number of members. Afterwards, Chang [3] noted how Hall Jr.'s work in [10] can be used to characterize marriage problems with unique solutions.

*The author would like to thank Stephanie van Willigenburg for her guidance and advice during the development of this paper. Furthermore, the author gratefully acknowledges the insightful advice and feedback from the anonymous referees. The author was supported in part by the Natural Sciences and Engineering Research Council of Canada [funding reference number PGSD2 - 519022 - 2018]. https://sites.google.com/view/btchan

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